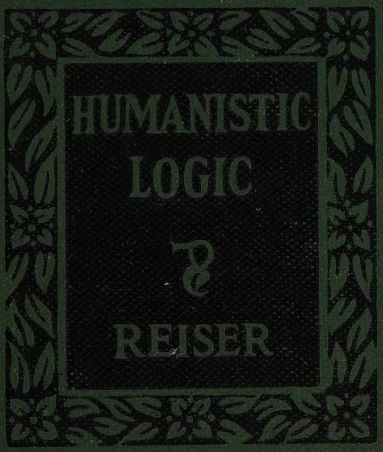
HUMANISTIC
LOGIC
&
REISER

In recent years there has been a reaction against the conventional presentations of the science of logic. Traditional discussions of the subject are now being criticized for their dryness and sterility. This, however, is not indicative of a growing skepticism concerning the value of the study. Indeed, it is precisely because educators feel that courses in logic, rightly presented, may have great value in developing the type of intellectual outlook which higher education ought to foster, that this criticism is made. It is because of its civilizing potentialities that educators insist that logic must be "vitalized" and "humanized."

The author has considerable sympathy with the revolt against traditionalism. But like many others, he feels that there is much of value in "formal" logic. It is held, therefore, that the way to improvement is not so much by way of subtraction from present practises, as through a transformation of the old to meet new interests. A greater coördination of the best of logical convention with modern science is also desirable. Accordingly, in the present volume an attempt is made to integrate logic and life by relating the thinking process to other human problems and conduct. It is, therefore, not so much a setting up of new standards, as a harking back to old. It restates the science in its original terms.

The work will be found a keen stimulant to classes in logic and related subjects. It represents a modern approach which is at once scientific and sympathetic.

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HUMANISTIC LOGIC
FOR
THE MIND IN ACTION

HUMANISTIC LOGIC

FOR

THE MIND IN ACTION

BY

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To
MY MOTHER

INTRODUCTION

THIS book aims at being something more than just another logic text. It attempts to broaden the scope of logic by relating the reasoning process to other human activities. The purpose of a course in logic is not conceived to be that of making human beings over into purely intellectual reasoning machines. The function of thought is held to be that of coördinating responses, of integrating human interests. It is held, therefore, that the function of a course in logic consists in showing individuals how to achieve a measure of intellectual synthesis, a unity of insight and outlook, in a world which is rapidly changing in its physical and social aspects.

The present volume is written under the definite conviction that before the logician can make persons logical he must get them interested in being as intelligent and as intellectually honest as they can be. Hence the first two parts of the book consist in an attempt to "humanize" logic, to relate it to the broader adventure of living in the modern world. Whenever an author attempts to do this he lays himself open to criticism from the experts, who apparently feel that the attempt to "popularize" a subject leads to superficiality of treatment. The author hopes that the present volume has not sinned in that direction.

By way of acknowledging personal indebtedness I would like to express my obligations to my teachers in philosophy and psychology in the Ohio State University. I wish also to express my thanks to Dr. M. R. Gabbert

for encouragement and to Dr. A. E. Avey, who has kindly read the manuscript and made many helpful suggestions. I need not say that any shortcomings of the book are to be charged against myself alone.

In concluding my statement of indebtedness I wish, by way of explanation, to point out that whatever element of human warmth pervades this book on Logic is to be ascribed to the influence of my wife. Her affection and understanding have provided a living example of the way in which cold logic may be tempered by feeling.

O. L. R.

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Part I

THE HUMANIZATION OF LOGIC

Chapter I

LIBERALISM AND LOGIC

ACCORDING to one eminent scientist the most important concept which the human mind has thus far formulated is the idea of progress. Other investigators might have their doubts as to whether this notion deserves *first* place in the list of the intellectual achievements of the human race. But probably no one would deny that this notion of progress—of social and intellectual advance—constitutes *one* of the major ideas operative in the creation and maintenance of the modern outlook and temper. Since this concept is such a very important foundation stone in the construction of the contemporary world it will be well to examine the idea in more detail.

What, then, *is* progress? In the first place, we observe that progress of any sort whatsoever—scientific, moral, or social—implies or presupposes *change,* a modification of the existing order. But, as we also soon discover, not all change is progressive. The change must be in a definite direction—towards some goal, near or far-off. Things may change without becoming better, without approaching closer to the ideal. In the animal kingdom, for example, some evolution has been the expression of a process of change in the direction of retrogression, at least when judged by familiar human standards. In that connection parasitism might be cited. Mere change, therefore, is not necessarily progressive. The change, to repeat, must be in the direction of improvement.

As a result of research in the fields of the social sciences it has become clear that social and political systems are and always have been subject to the processes of change and evolution. The *rate* of change may vary for different kinds of societies, but no society remains absolutely static for an indefinite period of time. As long as gregarious animals, particularly human races, have inhabited the earth social evolution has been a fact. But, as we have noted, mere historical change is no guarantee of progress. To be able to say that in making the change from primitive society to the present world society has progressed, is further advanced along the road to an ideal social and intellectual order, we must be able to show that our so-called civilized communities are in fact embodying in an ever-increasing measure those qualities and attributes which reason tells us are the essential characteristics of a more perfect social order.

The problem of stating what the requirements of a more perfect social order are is not an easy one. Experts are by no means agreed upon the matter. Fortunately for our purposes, it is not necessary for us to attempt to arbitrate the dispute. We need here only note that no matter what theory one may hold, in order to arrive at such a theory reason must be employed. Thought is the instrument by means of which such ideal formulations are outlined. And in order to put into effect the program of social action which such conceptions call for, intelligence must exercise the guiding hand in the process. Indeed, it appears to some that a static ideal of a perfect society cannot meet the demands of a society which is itself in process of evolution. That is to say, according to some thinkers an ideal society is one in which the individual is free to continue to evolve, to grow, to expand his interests, to give expression to his capacities—without limit. In this case a perfect society

is held to be a dynamic society, one that continues to change and evolve, while yet possessing sufficient order and coherence so that the possibilities for growth are not frustrated by group conflicts and excessive individualism.

From the foregoing account several features emerge as characteristic of social processes. In the first place there is a recognition of the inevitability of social change. The individual who would command the modern world to stand still is as foolish as the old king who ordered the tide to recede. Secondly, believing that man can in a measure control the direction of such change, there is the recognition of the need for intelligent guidance, for planned coöperation, so that the processes of change will not defeat the universal human ends which are held to be the condition for a healthy state of society. The problem here is one of achieving a dynamic equilibrium, of making our social institutions, no less than our intellectual outlook, sufficiently plastic so that we do not stand still and stagnate, but rather change our institutions and mental attitudes to meet new needs and environments, while yet maintaining sufficient contact with past modes of conduct to give meaning and direction—a sense of cultural continuity—to present policies and future plans.

And now, let us ask, what has this to do with logic?

The logician has no particular social program to advocate. He has no universal panacea for the ills of society. The logician's task is that of putting at the disposal of society—through the medium of education—those tools of reasoning which have been developed which represent and embody the proper technique for solving individual and social problems as intelligently as possible. Psychologists are of course not in entire agreement on the definition of *intelligence*. Some define intelligence as the ability to learn by experience. Others prefer to regard it as the ability to adapt one's self to new situations, to

solve problems. As is readily seen, there is no necessary opposition between these two formulations; for, as we will observe in more detail later, we solve the problems forced upon us by novel situations through the effective use of the accumulated knowledge which we have acquired in the past.

We see, therefore, that the problem of logic is the problem of making people intelligent, of making social action reasonable. The fundamental rôle which reason or thought will play—must play—in society if our social evolution is not to end in disaster is generally recognized by those who have considered seriously the problems of social evolution. From the numerous discussions of this subject which are now available we will select two presentations to illustrate and confirm the thesis which we are here setting forth.

The possibilities in the direction of reshaping the world through a reconstruction of our intellectual outlook was first pointed out in a way which commanded general attention by James Harvey Robinson in his book, *The Mind in the Making*. According to this historian the modern world demands a moral and economic reconstruction, and this can come only when genuine intelligence takes the place of what now passes for reason. Today society faces unprecedented situations, and this calls for unprecedented knowledge and attitudes. To deal adequately with this new world of perplexing problems what we need most of all is an open mind, Dr. Robinson tells us. We must bring our minds up to date.

This writer has the faith that if we could transform men's ways of looking at themselves and the world about them we could abolish a considerable part of the evils which now afflict society. He believes that the problems of labor and capital, race prejudice, political corruption,

and other vexatious matters would vanish if we could but transform our attitude towards the world.

To some disillusioned cynics this must seem like a rosy optimism. But Dr. Robinson is not alone in this faith. Bertrand Russell, who also shares this view, gives us some specific rules of procedure to bring about this intellectual reconstruction. Mr. Russell, an able logician, lays down the basic principles of his creed in his book, *Sceptical Essays*. He tells us: "It is undesirable to believe a proposition when there is no ground whatever for supposing it true." He then expands this statement into the following three principles: (1) that when experts are agreed, the opposite opinion cannot be held to be certain; (2) that when they are not agreed, no opinion can be regarded as certain by a non-expert; and (3) that when they all hold that no sufficient grounds for positive opinion exists, the ordinary man would do well to suspend judgment. These principles seem mild and innocuous, yet, Mr. Russell says, if they are accepted they will absolutely revolutionize human life. The opinions for which people are willing to fight and persecute all belong to the classes of propositions which Mr. Russell's type of skepticism condemns. Beliefs about politics and religion are held passionately but not reasonably. Beliefs about socialism, marriage, nationalism, and the like, are not the result of the activity of reason, but of prejudice, convention, or respect for tradition and authority, says Mr. Russell.

Messrs. Robinson and Russell take it as self-evident that the world would be a better place in which to live if men were more rational. Probably these eminent thinkers are correct in this. At any rate we have given the opposite plan, that of irrationalism, a long and thorough trial; we should do no less for rationalism. Perhaps an education so devised that it really taught human beings to

think critically and creatively might mold the world a little closer to a heaven here on earth. Perhaps we might terminate completely the short and simple annals of the poor. This is as reasonable a faith as can exist in an age which has shown all faiths to be non-rational.

As various investigators have pointed out, reasoning is a kind of feeling of one's way through the universe. Reasoning of a creative sort is an intellectual voyage of discovery along strange shores and over untracked oceans. The amœba explores its universe by projecting its pseudopodia out into the spaces about it. Here the enlargement of the environment is the result of tentative groping into the unknown. The scientist is also an explorer. He projects from himself his mental pseudopodia, those intellectual tentacles called hypotheses, which reach out even to the stars in their anticipations of further sensory experience. Perhaps we have here something more than a figure of speech. It is interesting to note that some authorities believe that in thinking a similar type of amœboid movement occurs in the nerve cells in the brain of the scientist as he thus ideally explores new worlds. If such be the case reasoning becomes in fact the experimental exploratory activity, only on a much more subtle and intricate scale, we observe in those lowly organisms, the amœbas, that feel their way by trial and error through the universe of micro-organisms.

But fortunately for us, we possess in some degree the faculty of intelligence, and this enables us to avoid some of the errors which would otherwise follow the trials. If we are to learn the rules of this game which enables us to feel our way about without making so many mistakes we must study logic. Entering now upon this domain, let us explore in some detail this country of the mind.

Chapter II

INCREASING HUMAN INTELLECTUAL EFFICIENCY

WE have it on the authority of Aristotle that man is a rational animal. The great Stagirite was called *The* Philosopher; according to Dante he was the "master of those who know." Admittedly, then, Aristotle was a rational animal. But clearly we are not all Aristotles. In the light of the behavior of many of the rest of the human race there is cause to wonder whether Aristotle was not guilty of the error of judging others by himself when he made his famous observation. This, at any rate, is the opinion of the self-elected intellectual aristocracy who contend that the optimism of Aristotle lacks adequate justification.

In support of the view that man's alleged power of reasoning is greatly exaggerated our pessimists might point to the fact that most of us use but very few of the hundreds of millions of neurones with which we are supplied. Indeed, in view of the fact that our enlarged fore-brain seems to profit us so little, some individuals might be inclined to agree with the suggestion of Morley Roberts that this structure may really have originated as a malignant overgrowth or tumor. Commenting upon this speculation, Professor W. M. Wheeler[1] remarks that this increase in the fore-brain may eventually be the cause for the extinction of man, who may some day be classed with the many races of animals that have perished of special overgrowths.

[1] In his volume, *Emergent Evolution,* in the New Science Series.

What is the truth in this idea that the brain is a useless overgrowth, a malign tumor, a cauliflower excrescence at the upper end of the nervous system? Much as one may respect this authority, it is doubtful whether this suggestion is to be taken seriously. The weight of the neurological evidence is clearly in favor of the view that the new brain is, in the words of the neurologist C. J. Herrick, the "specific organ of civilization." We may grant that we do not make use of as many of the cortical neurones as we might; we may admit that we exert no strenuous effort to combine these millions of individual nerve cells according to the laws of combinations and permutations to produce the practically infinite number of possibilities, which might conceivably underlie the production of individually novel and socially unprecedented ideas.

But does this mean necessarily that the neurones of our brain are useless, if not positively harmful? That this conclusion does not follow seems to be proved by the experiments of Dr. K. S. Lashley,[2] who has shown that the intelligence of rats is a function of the total mass of cerebral tissue. Experiments indicate that the destruction of cortical tissue in monkeys modifies their memory and capacity to learn, not in terms of the extirpation of individual areas, but in terms of the amount of tissue remaining in proportion to the total original amount of cerebral mass. That is to say, an animal with half its normal cerebral mass is less "intelligent" than it was before part of its brain was removed. The neurones here seem to exercise a facilitating effect on each other, so that the greater the total mass the greater the intelligence of the animal.

Of course here, as always, one can raise the question

[2] *Psychobiology*, Vol. II (1920), pp. 55-135.

of what light experiments on lower animals throw upon human behavior. Psychologists still admit—sometimes almost reluctantly—that there *are* differences between rats, monkeys and men; hence it is difficult to generalize about human beings from experiments on lower animals. But in connection with the present problem of the correlation of intelligence with nervous tissue the evidence points unequivocally to the truth of the view that man's superiority over the monkeys (or, more generally, the superiority of the big-brained mammals over the little-brained mammals) is made possible by the possession of that supreme synthesis of nature, the neëncephalon.

To remove the last doubt concerning the value of the central nervous system and to show with finality that the brain is something more than a knob to round off the spinal axis, the following story of an important biological event dating far back in the history of animal evolution may be told. Nowhere can we find a better illustration of the value of the cranium than in the story of the conflict between the brain and the belly, a conflict which occurred many eons ago. Here is the story as the biologist reconstructs it.

In the anatomy of the earthworm we find strands of nerve fibers running along the ventral side of the food tube, and connected across at each segment until the double chain gets to the head. When the cables get up to that part they form a ring or circle of nerve fibers surrounding the esophagus, or gullet. This gullet is part of the food tube, through which the food passes on its way from the mouth to the stomach. An eminent English biologist spent many years trying to find out how this type of nervous system developed into the nervous system which we now have, the system which belongs to the vertebrate or backboned animals. He finally developed the

theory which is here set forth in the words of Dr. William A. White.[3]

> These segmented organisms [e.g., worms] were dependent upon food, as all of us are; and the food became scarcer and scarcer so that they had to develop brains in order to devise methods of finding food when it was scarce. The type of organism of which I am now talking belongs at about the level of the spider. . . .
>
> The point reached by the spider, then, is this—he had to have brains, but the more brains he got the more they shut down on his food-tube, and if he developed too much brains he would starve to death; on the other hand, if he did not get food he would also starve to death, and he could not get food without developing brains. That is a difficult position to be in; and, as Professor Gaskell said, at that point in the history of evolution, living organisms, so to speak, had to decide whether, in the future, life would be a matter of belly or of brains. One or the other had to give way.
>
> Now an extraordinary thing happened. A new intestinal tract was developed outside the nervous system. The old gastro-intestinal tract ran right up through the bars of nervous fiber; but the central nervous system of vertebrates is developed on the back or dorsal side, and the gastro-intestinal tract lies ventral to it. There are some very interesting things to show what happened. The brain and the spinal cord are hollow organs; there is a series of ventricles, or cavities, in the brain, and a long ventricle running down the spinal cord. These are supposed to be relics of the old gastro-intestinal tract that was destroyed by the vertebrates.
>
> At the base of the vertebrate brain is the pituitary gland. Right through the center of the pituitary gland there runs a little channel, and that is the remains of the old esophagus of the spider. It is there today and the pituitary gland is made up of two distinct parts: one of these comes, during the development of the embryo, from the upper portion of the esophagus; and the other is derived from the nervous system. So we have within us today the trace of this old conduit, and we have to be thankful that the conflict between belly and brains was decided in favor of brains.

Therefore, without embracing the bizarre idea that the brain is a superfluous organ, at best serving only orna-

[3] "The Adjustments and Unity of the Organism," *Psyche,* July, 1926.

mental purposes, we may, if we are so persuaded, accept the conclusion that a large part of the brain of the human being is not of very great service to the average individual. Well supplied with cortical neurones as we are, few of us use any great proportion of them. Some theorists have attempted to estimate this proportion: one favored ratio sets the amount of cerebral tissue which we actually use as $\frac{1}{10}$th of the total brain matter. It is difficult to see how this can be established, but even accepting this figure as fairly reliable, this does not prove that nature has wasted the other $\frac{9}{10}$ths. The fact remains that man is more rational than any other animal, and this power to reason is certainly dependent upon something in the nature or structure of the brains which human beings inherit.

Today all scientists are agreed that the dividing line between man and the lower animals is much less clear and distinct than the older philosophers and psychologists supposed. Anyone who has been influenced in his thinking by evolutionary views must inevitably come to consider the question of whether man possesses any quality or ability not possessed by the animals more closely related to him. The tendency seems to be to regard differences between man and the other animals as a difference of degree. If the human mind emerged from the animal mind, as some scientists hold, then perforce we must attribute to animals, only in lesser degree, the possession of qualities which attain such a high development in man. Here science and popular thought can agree, for it is hard for a dog lover not to attribute to his pet dog the qualities we ordinarily attribute to our fellow human creatures. We may, therefore, sum up the modern developments in this field in the statement that man is no longer thought of as the purely rational creature of the older intellectualistic psychology, while parallel with this level-

ing down has gone the elevation of animals to a position closer to the human species, so that the "brutes," the "dumb" animals, are now thought of as being more intelligent than the older theorizers would admit.

Man, therefore, is not to be defined as the thinking animal, *par excellence*. It would be more advisable to say that man is an animal who can think *better* than other creatures. What does distinguish man from the other animals (so far as the power to reason is concerned) is the fact that man is conscious of his thinking as such. That is to say, man realizes sometimes that he makes errors. In a word, man is an animal who can *think about thinking*.

Philosophers have coined the world *epistemology* (from the Greek word meaning knowledge) to designate this study of the problem of truth, falsity, and error. If this ability to discriminate between truth and error is confined to the human race then we can truthfully say that man is distinguished from other animals, even the anthropoid apes, by the fact that he is an epistemological animal. We will return to these questions of the comparison of human and animal thinking and the criteria of truth and falsity at later points. For the present we must follow up the assertion that man is an animal who can think a little bit about thinking.

This is not the whole story. The most important thing for us is the fact that by thinking about thinking we can improve the quality of our thought. This is only another way of saying that man can learn by education and self-criticism. No animal possesses anything like our ability to learn by experience. What the upper limit for the human race in efficiency in reasoning may be is hard to say. That there must be such a limit is obvious. What the economist calls the law of "diminishing utility" has its analogue in psychology in the law of "diminishing intel-

lectual returns." That is to say, we, as individuals, must eventually reach the point in our mental development where *each added increment of intellectual effort fails to produce as much by way of returns for the effort as the preceding amounts of study*. But as yet few of us have reached this physiological and psychological limit of improvement. Let our efficiency be what it may be—high or low—it follows that any method which would elevate this level of intellectual performance by 50%, 10% or even 5%, would be a method worthy of study.

If, then, man is so docile and plastic, we may well ask why it is that our critics are so pessimistic, doubting whether human stupidity has declined or the human race improved mentally. To be sure, it is possible that these critics are depreciating the intelligence and the achievements of the great majority merely in order to elevate themselves (in their own eyes) by comparison. But even making allowances for this, no one who has observed contemporary social performances can deny the low standards which obtain in popular thought. If this is the case, we may well ask: Why is this so? Why is it that Matthew Arnold felt constrained to pray so fervently, "More brains, O Lord, more brains!"?

Part of the explanation of this was given by John B. Watson when he pointed out that the environment lets us live at a low level of efficiency. Public opinion is tolerant where the sins of the majority are concerned. Irrespective of the final decision concerning the debatable problem of the inheritance of mental traits, we can all agree that we could all think more and better if we were forced by social pressure to do so. From a pragmatic point of view Watson is justified in insisting that education could function much more efficiently than it does.

It has been said that necessity is the mother of invention. Undoubtedly this is so. But, as someone has

added, if necessity is the mother of invention, curiosity is the father. With respect to inventiveness in the field of ideas neither home education nor the school has been sufficiently stimulating. The tendency seems to be to stifle curiosity by asking children not to ask so many questions. As a result we are suffering from what might be termed intellectual anemia. This has not yet developed into the pernicious type, but unless we take steps to preserve our vitality by proper nourishment and exercise we cannot be sure that our troubles will not become worse.

But after all, someone asks, is the damage done by what you call intellectual anemia as serious as you have suggested? No one can answer that question with certainty. We cannot compare the present status of man and society with that ideal society which might conceivably have resulted if man had learned to use his brains fully and efficiently long ago. Nor is it possible at present to anticipate the characteristics of a future society through speculations concerning the probable direction which human evolution may take, for evolution sometimes takes new and unforeseen turns. It is conceivable that the reasoning of the superman (supposing for the moment that this mythical individual is on his way) may transcend present modes of inference in ways which we cannot now discern. Perhaps a future Aristotle may do for logic what the non-Euclidian geometers have done for their branch of mathematics. To consider such possibilities will be the purpose of a later chapter (Part II, Chapter VII).

Chapter III

THE HUMANIZATION OF LOGIC

IN the preceding pages the reasons have been set forth for believing that courses in logic have not functioned very effectively in promoting human thought. Education in general has not improved the amount and quality of thinking in any considerable degree. This is illustrated by the fact that most college graduates still prefer a game of golf to the game of playing with ideas. The admission of this fact, however, does not mean an admission of defeat. It sets forth a challenge to our present educational system. And to insist upon this need for higher standards is surely one of the duties of the discipline of logic. If, therefore, the foregoing and following chapters, as a result of comparison with other texts, appear out of place in a book devoted to the study of logic, it must be remembered that they are written under the influence of the conviction that before you can make men logical you must get them interested in being intelligent.

The uninviting character of logic, as traditionally presented, was in part a result of the influence (consciously or unconsciously exerted) of the doctrine of *formal discipline*.[1] According to this educational theory it is the business of instruction to discipline the mind. The best way to accomplish this, it was believed, was to compel the student to master a subject—such as logic or mathe-

[1] The doctrine of formal discipline was associated with the educational theory of "transfer of training." We will have occasion to examine this theory in more detail in the next chapter.

matics—by exerting an effort. It was supposed that if the subject were naturally interesting to the individual, so that no great effort was demanded, no disciplining of the mind would result. Hence the subjects most favored in the older classical cultural theory of education frequently became distasteful.

In recent years there has been a reaction against this view. The attempt is now being made to make education as inviting—even as easy—as possible. In line with this tendency subjects like mathematics, which some pupils dislike, are being abolished as required subjects. Perhaps we are now going to the opposite extreme of sugar coating education. Since the time of J. J. Rousseau, who expounded the educational theory that the best way to educate a child is to let him alone, we have tended to let children choose their school subjects according to their interests, whatever they may be. Thus if a child prefers manual training or cooking to the classics or history he is now much more likely to be permitted to study what he is interested in than he would have been under the older scheme of education.

In order to orient our own treatment of logic to this dispute between what we might call the older monarchical and the newer *laissez faire* theories of education, let us point out that, whatever the phrase the "humanization of logic" may mean, it does not mean making logic so shallow and devoid of substance that anyone can master the subject without the expenditure of any effort. The danger of many so-called "popularizations" is that they lead to superficiality of treatment. But on the other hand, many subjects can be made more interesting than they have proved to be, if they are related in various ways to the world in which we live. Logic is in as good a position to take advantage of this fact as any other subject. It is true that if you can make a goal sufficiently at-

tractive to an individual he will be willing to exert himself to arrive at that goal. In this respect the pedagogy of the "new" logic seems to resemble more the "project" method of teaching than it does either the doctrine of formal discipline or Rousseau's views.

In the past text-books of logic have occasionally been inconsistent. While writers of logic books warn us of the necessity of discovering the assumptions we make in our thinking, the writers of these same books have sometimes failed to live up to their own dictum, especially when they fail to make explicit what the assumptions are which provide the justification for a course in logic in a university curriculum. The statement is usually made that the function of a course in logic is to teach students to think correctly. Some of the characteristics of good thinking are said to be freedom from bias, the ability to exclude the influence of emotional disturbances to thinking, and so on. And yet very few books tell us what the proper relation between reason and emotion is. Does this mean that the student of logic who wants to be a clear thinker should suppress or eliminate his emotional make-up? Logic, along with other studies, is supposed to fit us for adjusting ourselves more intelligently to the universe in which we live. But if logic is to help fit us for life, what is the proper relation between reason and habit? How much respect ought we to pay to authority, tradition, and human sentiment? The prevalent belief that emotion is hostile to accuracy and fertility in scientific research is part of the doctrine that reason and emotion are natural enemies, and that truly logical thinking is utterly dispassionate reasoning. But the actual fact is that many brilliant discoveries in science are the result of a type of psychic motivation which resembles in some respects the obsessions of an insane man. If this is true, are we, in the interest of "dispassionate" and "unbiassed"

logical thought, prepared to depress scientific creativity by weakening those powerful driving forces which spur men on to new discoveries? Here again logic texts tell us little.

All this may be summed up in the statement that a really adequate treatment of logic must be much broader in its scope than the traditional treatment has been. Logic must be expanded so as to become the science of the whole mind as it functions in adjusting itself to the physical, social and intellectual environments. Logic, in other words, deals not alone with thinking "as such," but also with thinking as related to the other functions and capacities of human nature.

The different sciences which take mankind for their study have a tendency to split up the human being and deal with him in terms of abstractions. Under the influence of this methodology we have come to think in the various sciences of the "economic" man, the "political" man, the "religious" man, and so on. But in reality it is the same human being who enters into these different relations. Whatever the justification for these types of specialization may have been in the past, the time has come to deal with the integrated man as he strives to adapt himself to the diverse and often conflicting groups and interests of the modern complex world. In reality each human being is an inhabitant of three worlds. There is (*a*) the inner world of consciousness, private to each individual and apprehended by us through our introspective experiences; then (*b*) there is the physical world of objects, the external environment, to which all of us, by virtue of the fact that we are also physical objects, must adapt ourselves; and (*c*) there is the social world of human beings of which every individual is normally a member. These worlds overlap and interpenetrate, so that it is sometimes difficult to tell where one leaves off

and the other begins. Under such conditions the adjustments we make are likely to interfere with each other. In the light of these considerations the problem of reason and intelligence is that of averting the threatened chaos which menaces us. Our problem is to live effectively in all three universes.

As yet no course exists which can teach us how to achieve the thorough and final integration which would result if all conflicts and disharmonies were abolished. Such a completely worked out synthesis of knowledge and life is impossible at present. But the pursuit of the goal is none the less worthy and essential. The field of logic comes as close as any to that study whose business it is to assimilate the data from the separate fields of (*a*) the natural sciences, (*b*) the social sciences, and (*c*) psychology. Logic is the intermediate science which bridges the gap between the outer and the inner worlds. As a branch of philosophy it must occupy an important place in intellectual integration.

If logic is to assist in this task of mental reorganization and synthesis, it must be more willing than it has been in the past to bring together the knowledge gathered from these separate fields. As thus defined, the function of logic comes close to what now goes under the title of the philosophy of science. In line with that conception, we will have occasion to introduce material not ordinarily found in the traditional treatments of logic. From this point of view, for example, the customary separation of logic and ethics has no justification other than that of its pedagogical necessity. Good ethics must be logical, in the sense of being able to stand the acid test of reason; and, on the other hand, good logic must be ethical. Morality which is not reasonable is not morality, and reasoning which is not intellectually honest is not socially valuable. Really good thinking aims to be both logical

and ethical. If, for some reason or other, these two, logic and ethics, are incompatible, the sooner we find it out the better.

In accordance with this view, we will discuss in more detail this matter of the interaction of logic and ethics in Chapter VI of this Part.

Chapter IV

CAN MEN BE MADE LOGICAL?

IN the previous chapters we have taken the position that the quality of human thinking could be improved, provided we employed the right type of educational technique. The time has now come to consider in more detail the complexities involved in this view. In order to get our problem before us in a more definite form, let us narrow our discussion to the question of consistency in thinking. This is not to say that the problem of being intelligent is identical with the demand for consistency. But there is a sufficiently close association so that we can discuss the problem of how to be reasonable and intelligent in terms of the problem of how to secure consistency in our various sets of beliefs.

The first problem we have to consider is this: Is consistency in thinking really desirable? The problem of whether it is *possible* to induce people to be more consistent needs to be considered only if and after we show that consistency is desirable, a jewel to be highly prized. The view that consistency is not desirable is apparently, in practise if not in theory, the view of not a few persons. Thus, to give one illustration, a scientist friend of the author states that he enjoys going to church on Sundays and living for a while in the comfortable idea that there is a beneficent Father in Heaven looking out for His children, even though, as this scientist states, he believes that this practise is out of keeping with his beliefs as a scientist, professed on the other days of the week. This same individual affirms that some of the greatest bores he

has ever met are persons who are utterly and hopelessly consistent.

This same view has been expressed by a number of literary men. Oscar Wilde states that consistency is the mark of a mediocre mind. But, in reply, may it not be pointed out that his own tragic end is an ironical though fitting epitaph to that type of logical scepticism? May not his own *De Profundis* be a belated attempt to reconstruct a morally and intellectually consistent interpretation of life? Another iconoclast of human thought, Friedrich Nietzsche, declares that convictions are prisons. But is not this prophet of poetic paradoxes falling victim to his own dictum? Is this proposition not itself a conviction which imprisons him? Walt Whitman, in his cavalier manner, exclaims: "Do I contradict myself? Very well, then, I contradict myself!" Emerson has also said something similar.

But we may ask in reply: What scientist, social or physical, could with serious face utter such logical heresy? Moreover, it can be demonstrated that such a view is self-refuting. Venders of inconsistencies should at least grant that they ought to be consistently inconsistent. And as Josiah Royce pointed out, when we come to propositions so fundamental that the very attempt to deny them reasserts them (the denial that consistency is a valid ideal is put forth as a consistent view), we are pretty close to the rock bottom of human belief. The simple fact is that, in our saner moments, we all recognize that consistency is a desirable quality of ideas and actions, so that any means that can be employed to increase the measure of consistency ought to be encouraged. An intelligent solution to our problems, scientific, moral, and political, can come only as a result of clear, consistent, and hard thinking. In a democracy the widest extension of consistency is pre-eminently desirable.

It will therefore be taken for granted that consistency is a consummation devoutly to be sought. We must next consider the question of whether it is *possible* of attainment in greater measure. To what extent is it possible for the masses of individuals constituting our democracy to achieve consistency? Some theorists have raised a similar question in connection with intelligence. In the light of their analogous problem we may rephrase the question as follows: Can consistency be acquired through the educative process, or is the possibility of being consistent determined by innate factors? This, we see, is nothing other than the standing question of whether the environment or the heredity of an individual is most important. If, as seems likely, there is a positive correlation between intelligence and consistency, the answer we give to the question of whether we can increase the amount of consistency in individuals will be related to the answer we give to the question of whether intelligence is innate or a product of training and education.

Some eugenists, emphasizing the importance of the hereditary factor, have tended to minimize the importance of education as a means for improving the human race; they hold that improvement can come mainly from raising the level of native capacity by selective mating, by encouraging a higher fecundity in the intellectually well-endowed while at the same time discouraging the birth rate among imbeciles, idiots, feeble-minded, and even the morons. The environmentalists, on the other hand, stress the importance of education in the creation of human attitudes and abilities. Since the entire matter of the value of the teaching of logic is involved in the problem, it will be well for us to examine this matter more thoroughly.

If it is true, as the hereditarians assert, that men are either intelligent (and logical) or unintelligent (and

illogical) by nature (original endowment), it follows that you cannot make a man logical by nurture who is illogical by nature. On the other hand, it may also be argued that if a person possesses high mental ability by virtue of his inheritance he will not profit very much by a course in logic. He will not need it. In either case courses in logic seem superfluous. If you have intelligence you do not need someone else to teach you how to be logical, and if you do not have intelligence logic can't teach you very much. Such a view has indeed been accepted, even by teachers of logic, who, presumably, continue to teach logic, not because they believe they are helping the world progress mentally, but because they have to make a living. One logician summed the situation up in these words: a study of logic will no more make you logical than a study of masses will make you heavy!

The foregoing line of argument has been summarized in the form of what is called a dilemma. Part of such an argument consists in what is called a disjunction—we are first offered several alternatives, the horns of the dilemma, and then there is an attempt made to show that certain consequences follow. In the present case we are offered the alternatives: either ability is inherited or it is acquired (not inherited); but if the first is true, as the hereditarians assert the empirical evidence proves, then a course in logic is either unnecessary or ineffective. By way of criticism of this argument, which is so antagonistic to the spirit of our previously announced doctrine, we may point out, first of all, that we must be sure that our disjunction is complete, i.e., covers all possibilities. As a matter of fact it is doubtful whether the alternatives offered us cover all possibilities. It may be, for example, that intelligence is a product of two variables, heredity and education. In the second place it must be noted that this form of argument is never very convincing. As we

will see later,[1] it is frequently possible to formulate a counter-dilemma which attempts to prove the opposite of what was asserted by the original dilemma. In the present case we may formulate the following proposition: no matter how much mental ability is determined by inheritance, you must have the proper environment to bring out these capacities. If reasoning ability is innate you can therefore profit by the study of logic, and if ability is not innate, but a reflection of the type of training and education one has had, a study of logic is still of value in contributing to the production of intelligent thought and action.

It must be understood that we are here neither attacking nor defending the doctrines of the eugenists and the environmentalists. We are merely trying to show that in terms of either doctrine logic still retains its values as a study.

Another value of logic arises out of the following circumstance. Many persons who are critical, who strive for consistency in their beliefs, can feel the invalidity or inconclusiveness of an argument. And yet, unless they have studied logic, they are frequently unable to put their finger on the precise source of difficulty. One may feel that an argument is fallacious, but unless one knows the rules and the names of the fallacies to which human thought is subject, one is unable to specify what the fallacy is. To be able to refute another's argument by showing that it involves the fallacy of *begging the question, false analogy,* or the fallacy of *undistributed middle,* or whatever the fallacy may be, is a great convenience to one who has to work in the field where accuracy of thinking is essential.

It is a curious fact that those who deny the innate equality of man, who believe that men inherit differences

[1] In Part III, Ch. V.

in ability, and who therefore see in eugenics and not primarily in education the hope for the improvement of the race, are, perhaps in spite of themselves, compelled to place a considerable faith in education. A program of action for eugenics carries with it the enactment of certain laws by the state (such as sterilization laws, medical examinations before marriage, etc.), but such laws cannot be passed until legislative bodies recognize the need for such laws. This necessitates a program of educating the legislators, for if the people who make the laws cannot be educated to the point where they will pass intelligent legislation then the program for eugenics cannot be put into effect. In other words, whether we like it or not, we are committed to the view that it is possible to raise the level of thinking in the average person, including legislators.

We will grant that at the present stage in the evolution of human thought in science, religion, politics, and the rest, no man can be completely consistent. But it is also clear that many could be much more intelligent and logically consistent in their beliefs than they are at present, if they would set for themselves ideals of more rigorous thought. Aside from whatever limitations heredity may impose, the chief causes for the lack of consistency are mental laziness and the tendency to regard certain interests and activities, such as religion, as lying outside the field of reason. Persons who are engaged in the promotion of religious ideals may be inconsistent in their beliefs, not because they are inherently unintelligent or lazy (the reverse is frequently true), but because religion is commonly supposed to be concerned with spiritual matters. In short, religious and political beliefs may be inconsistent with themselves, and with other beliefs which the person holds, because human beings are

not forced by social pressure to reason out their views systematically.

The bearing of these conclusions on the problem of transfer of training may now be outlined. The doctrine of transfer of training was based on the theory that the human mind could be analyzed into distinct "faculties" or "capacities." The three main faculties were held to be *thinking* (reasoning), *feeling* (emotion) and *will* (volition). It was supposed that the faculty of reasoning was like a muscle: it could be strengthened by use. Mathematics and the classics, the backbone of the classical cultural curriculum, were believed to furnish the best material for this disciplining of the mind. Once the mind (reasoning in particular) had been trained and disciplined by mastering these subjects it was more proficient in functioning in other fields where reasoning was required (as in politics), because, it was believed, this training was transferred to the new field. This is the essence of the doctrine of formal discipline and transfer of training.

It is now pretty well agreed that faculty psychology gave us an inadequate analysis of the human mind. But does the abandonment of faculty psychology permanently destroy the doctrine that mental training can be transferred from one subject to another? The answer cannot be given categorically. Educators now seem to agree that in so far as one subject is similar to another subject to that extent proficiency in the original subject can be transferred to the new subject. This does not mean that a sailor who has trained his powers of visual perception on the seas would necessarily be more competent in a research laboratory requiring microscopic or telescopic observation. But it does mean that in so far as physical science, for example, requires a knowledge of mathe-

matics, to that extent previous training in mathematics is of value to a physicist. The individual can transfer his mathematical training to the new field.

And now where does this leave us with respect to the problem of the value of the study of logic? It may be argued that training in the use of logical methods can be transferred to any field whatsoever if the individual is interested in doing this. If it is true, as logicians assert, that logic is the study of the methods and forms of reasoning employed in any field where knowledge is possible, then it seems to follow that training in logic will be of great value to the individual who has the ingenuity to make use of that training.

But we must not oversimplify our problem. This question, as the well-informed will recognize, eventually involves the disputed doctrine of "general intelligence." We will have more to say about this in the next chapter.

Chapter V

HEREDITY, ENVIRONMENT, AND INTELLIGENCE

IN the past progress in science has been most remarkable in the sciences dealing with the external world. The power of scientific knowledge has been most abundantly demonstrated in physics and chemistry. Practical inventions, which are for the most part an application of physical principles, have largely been responsible for the Industrial Revolution, which has given us the factory, quantity production, and our present means of transportation and communication.

On the theoretical side progress in the physical sciences has been no less remarkable. As one scientist has put it, physics in the last two decades has been running wild. Such doctrines as the reciprocal convertibility of matter and energy, relativity theory, the undulatory theory of the electron, cosmic rays, and other creatures of the scientific merry-go-round have passed before us. We have gotten into the frame of mind of expecting the unusual, and nothing seems to surprise us any more.

There are those who assert that this rapid growth in the theory and practise of physical science is destined to be duplicated in the younger biological and social sciences. Much has already been done in the last several generations. But apparently we are in for even more remarkable advances in the domain of the organic world. Discoveries in this field promise to rival in interest and importance the wonders of physical science.

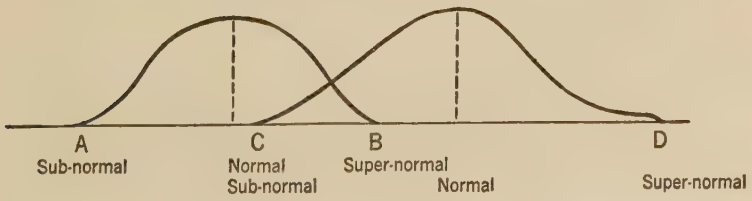
But is the human race any better off for all these scien-

tific discoveries and inventions? Is our thinking any clearer, are our beliefs any more critically formulated? Is mankind any happier for sanitation, newspapers, and rapid transit? Many observers fail to see that we have advanced intellectually in any noteworthy degree, because of these scientific advances. We have radios, to be sure, but what good are they if no one has anything important or valuable to say over them? Of what benefit are rapid transit facilities if what we have to do, and actually do get done more quickly, is hardly worth doing? What does it profit a man to go to more places, see more things, cover ground faster, if this does not enrich his life qualitatively? Do the accomplishments which we pride ourselves upon really make our civilization great? These are some of the questions which contemplative individuals, Orientals like Rabindranath Tagore, for example, would like to ask us.

Those who put their faith in the future of the human race because they see in science, working through education, the messiah of humanity, usually point out that as yet we have hardly made a beginning in the application of scientific methods to social processes. These futurists believe that the same tremendous strides will characterize social advance as soon as psychology, sociology, and political science begin to make their contributions to society. According to such men as James Harvey Robinson, the next great step in human advance lies in refashioning the human mind and society in the light of a better understanding of their nature and workings. Eugenists also have this same crusader's faith that the future of the human race rests with the social scientists.

It seems strange that those who argue these views with such religious fervor should fail to consider some very elementary and obvious problems which arise. Let us

suppose that by an intensive application of education and eugenics we could push up as far as you please the mental level, the general intelligence, of the members of society. Does that bring us any closer to Utopia? If we push up the general intelligence in the same proportion in all human beings, would we not get the same relative distribution of capacities which we now have? Would we not still find in society the morons, the average people, and the geniuses? Probably we would. The only difference that would be brought about would be that the type of performance of those who are now considered geniuses would be considered the performance of average individuals; the performance of those who are now considered average individuals would then constitute the performance of morons; while the performance of morons today would in the new society be the performance of feeble-minded persons. That is to say, the terms "normal," "supernormal," and "subnormal," are relative terms, so that a person who is normal (average) for one group might be sub- or supernormal normal with respect to another superior or inferior group. This, presumably, is the meaning of the statement that it is better to be a big fish in a little pond than a little fish in a big pond. What we do when we push up the general intelligence in equal proportions in all the members of society is merely slide the whole curve further along, without altering the proportion of the average people, who, like the poor, are always with us. This may be represented graphically in the two curves of the following diagram:



One wonders whether a society is any better off because the curve of ability has been shifted? Do eugenists mean by improving the race merely this, or is an improved society one in which we have a greater proportion of geniuses and a smaller proportion of morons? But if this is what the racial perfecters mean, can any in this new society be called geniuses if all are persons of superior ability? Is this not the night where all cows are black?

These are relatively simple questions. Further reflection on the matter reveals additional complications. The curves we have drawn are (hypothetical) statistical distribution curves. They are supposed to represent the distribution of human abilities in society as a whole. That is to say, if you take any specific ability, such as musical or mathematical ability, and place an individual with respect to others in the degree of proficiency he possesses in this ability you will find that the average individual (or the great majority of persons) is (or are) just mediocre, a few are highly proficient and a few show up poorly. If you do this for all (or at any rate most) fields of activity the same general distribution will result. If we combine these curves of abilities we will find that most human beings are just average in most activities. Abraham Lincoln summed up this fact in his statement that the Lord must have loved the common people because he made so many of them. The "common man" is so common because that is the name we give to those of whom there are the most.

But now let us note this fact. While it is true that most people have mediocre ability in most fields of activity, nevertheless in some subject each person probably possesses exceptional capacity. The statement that some "shiftless" individuals are "good for nothing" is hardly ever true. In other words, all are geniuses or near-

geniuses at a few things, mediocre at most things, and morons or worse at a few other things.[1] The reason this fact is not more generally recognized is that society has placed a high value on certain types of activities and discounted others, so that *men are called geniuses only if they possess the type of ability which has a high social value.* But psychologically (or biologically) the same fact, whatever it is, which is responsible for the normal curve of distribution for the abilities of individuals in society as a whole gives us the same type of distribution in the individual members of that society.[2] The fact that all men are well endowed in some respects—if it is

[1] This view has been argued for by Professor C. Spearman in his theory of the nature of intelligence. Professor Spearman's doctrine is called the "theory of 'g.'" This theory asserts that the measure of every different ability of any person can be resolved into two factors, the one of which, the "g," is always the same, and therefore constitutes the "general" factor. The other factor, the "s," is independent and specific The one which is always the same for an individual is closely connected with what is known as intelligence, and is supposed to measure some sort of "energy" derived from the whole cortex. This factor is held to be very important. In science and philosophy the "g" plays the same rôle. But in certain crafts requiring specialized skill success does not depend so much on the general factor, "g," as upon the specific factor, "s." According to Spearman, this cortical energy can be concentrated in the special cortical areas to which the "g" may be made available. The "s" measures the respective efficiencies of the different parts of the brain in which this energy can be concentrated, as in an "engine."

The empirical results which Spearman and his students have obtained appear to be in harmony with the theory. But the mathematical handling of the statistics which Spearman employs has been criticized, so that the doctrine is still unverified.

Spearman's doctrine is expounded in his paper, "Some Issues in the Theory of 'g' (Including the Law of Diminishing Returns)," *Nature,* Vol. 116 (1925), pp. 436-439, and in his volume, *The Abilities of Man.*

[2] There is much talk about the "law of averages." The grading system in many universities is based on the doctrine that of those who pass a given course a small percentage must get a high grade, a much larger percentage of the class must get an average grade and a smaller percentage get a passing mark. I have never seen a good rational justification for this practise. The fact that the empirical results are usually distributed in accordance with the normal (bell-shaped) curve does not provide a theoretical justification for the continuation of the practise of grading. What we actually do is construct our examinations and mental tests in such a way that the results will distribute themselves according to the normal curve, and if the results do not come out according to our preferred curve of distribution we say that the test was not a good one and then doctor it up to give the results we expect.

only in the ability to avoid work—and not in others should make us more modest, for we then recognize that the other man's stupidity (in those abilities which we value highly) is probably compensated for in the possession of other capacities on which we (and society) do not place a high value.

Eugenists beg the question when they talk about the danger of society being swamped by morons. They are implicitly assuming that society will continue to respect current judgments about those capacities which are socially valuable and those which are valueless. Moreover, when they tell us that the general level of abilities must be elevated they must be prepared to tell us what sorts of abilities they are talking about. Do the eugenists mean that one should become as much of a genius in every field as it is possible for one to be? Or do they think that in our selective breeding we should exercise occupational discriminations, showing a preference for poets, or scientists, or mechanics, and so on? Assuming for the sake of argument that we can produce individuals according to blue-print specifications, what is the proper percentage of artists, merchants, thieves, politicians, preachers, and the like?

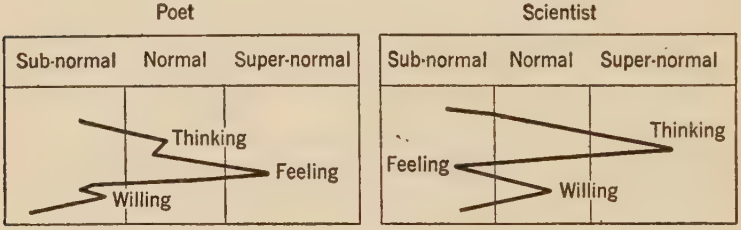
Unfortunately for this program, there are those who think that it is undesirable to elevate all human beings, either equally (i.e., in all abilities) or according to some professional discrimination. Dr. H. H. Goddard (who coined the term, *moron*), for example, tells us that we need the moron, and that if all men became intellectual aristocrats none of us would want to do the manual labor which is essential to society as now constituted. Others doubt whether there is room at the top for all. In his study, *The Twilight of the American Mind,* Walter B. Pitkin maintains that a great increase of high-grade intelligences will precipitate a crisis, since there is not now

enough suitable work for our "best minds." Still others, including the brilliant English biologist J. B. S. Haldane, think that society is moving in the direction of a social organization resembling ant societies, where a rigid caste system is practised. In such a social order the vast majority of unintelligent workers would be controlled by a small group of intelligent rulers who do all the thinking that is necessary. No matter which of the above possibilities should come to be realized, a serious set-back to the eugenic program would result. We are not arguing that any of these possibilities will become actual, but merely pointing out that those who are attempting to improve the quality of human performance must consider some intricate questions before they can claim to have formulated a reasonable social philosophy which they can ask society to put into effect.

And now we must consider the question which the reader has undoubtedly been waiting patiently to raise for some little time. It will be asked, but how about your own position? Are you not arguing for a view which is as much open to criticism as the views of the eugenists and the environmentalists, both of whom are also trying, in their own appropriate ways, to elevate the quality of human performance?

In order to answer this question we shall have to amplify some statements which we have made in preceding pages. When we have pointed out that the quality of performance varies with the individual we have merely pointed out what is common knowledge. Individual differences are a fact. Such general terms as intelligence cover a multitude of different types of intelligence, and human beings differ in the amounts of the different types of intelligence which they possess. In most abilities the average man will be normal, as we have stated. The number of types of these abilities will vary according to

the type of classification of abilities which is employed. Old-fashioned faculty psychology, as we have seen, divided the human mind up into three types of functions: thinking, feeling (emotion), and will (volition). The possibilities which might result from the different combinations of these faculties in different individuals is larger than the number of abilities. A poet, for example, might possess supernormal emotion and less intelligence, while a scientist might possess supernormal capacity to think and subnormal capacity to feel. And so on. We might summarize the situation graphically in the following way:



This is one type of classification of abilities used merely for illustrative purposes.

Another type of analysis of abilities, one which Professor E. L. Thorndike has suggested, breaks up intelligence into several different types, as follows:

The Main Types of Intelligence:
(A) Mechanical Intelligence—Muscular Skill, etc.
(B) Social Intelligence—The "Good Mixer," etc.
(C) Abstract Intelligence—Logical Ability, etc.

If we rate each individual as being good, bad, or indifferent in his possession of these types of intelligence we would get a fairly large number of classes of human types. We could then undertake an analysis of the types of human occupations and professions, determine the types of abilities they require for high efficiency of perform-

ance in that field, and then try to fit human types and occupations together. *Job analysis* and performance tests set for themselves this problem of finding the right man for the right job, so that we don't try to put square pegs in round holes and round pegs in square holes. This indicates the practical value of this type of study, worth noting in passing, even though our own problem is of a more theoretical nature.

Without assuming that either of the foregoing types of analysis of human nature is correct, we may use them for illustrative purposes in clarifying our own point of view. In the first place, it ought to be clear that we do not want to make all people alike in all respects. To use the old maxim, it takes all kinds of people to make a world. A society in which all human beings had become mechanically efficient—at the expense, perhaps, of other abilities—would work if we needed no other types of individuals. This simple observation, which might be applied to any other type of special ability, shows us that we do not want to push up the capacities in all individuals to the point where they are all equally proficient in all activities. *But there is no reason why we should not push up the other types of capacities which are of social value in every individual.*

But is there any quality or capacity, the possession of which would benefit both the individual and society, and in the perfecting of which the individual would not render himself less proficient in some other way? One of the capacities which comes close to meeting these specifications is what the psychologists have called "general intelligence." Without trying to make all individuals mechanics, or artists, or politicians, or scientists, or what-not, we can agree that every one of the individuals in any of these professions *would benefit by the possession of general intelligence,* over and above whatever special

type of ability may be required for his specific vocation. Clergymen, doctors, politicians, and even unskilled laborers who are unintelligent are not fit for *any* profession. We have already set forth our reasons for insisting that the environment be made as conducive as possible to the development of thought, critical and creative, so that we need not elaborate further on that theme. We may point out, however, that this way of stating the position leaves such questions as the inheritance of acquired characteristics and the dispute between the hereditarians and the environmentalists to be settled entirely by what experiment may eventually show to be the correct viewpoint in such matters.

Finally, the question is asked, Is there any evidence that there is anything in human nature which corresponds to the term "general intelligence"? May this not be a mythical entity, like the "caloric" and "phlogiston" of early chemistry? This is another debatable subject. Speaking for myself only, it appears that some persons possess a good bit of what this term implies. These people, of whom Aristotle and Leonardo da Vinci are supreme illustrations, have a power of comprehension, of organization, generalization, and a versatility in a variety of fields. They apparently are able to transfer their mental discipline from one subject to another. But we have already set forth our view on this subject of the transfer of training in logic, so that anything further would be by way of repetition. We must keep in mind that in the present stage in the development of psychology definite conclusions are impossible, so to avoid dogmatism we will make no further claims. All we can say is this: let us try to make people as intelligent as they can be made and then see what happens. Perhaps in order not to retrogress we must push forward. The only choice we can make is to push onward, even though we can only hope that this onward is forward and upward.

Chapter VI

THE MORAL OBLIGATION TO BE INTELLIGENT

THOSE nations committed to what is called the democratic theory of government recognize that every person has an equal right to attain happiness in this world. According to some social thinkers the function of government is negative in character: it is to remove the obstacles which would otherwise prevent the realization of that happiness. These are the philosophers who argue that that state governs best which governs least—the state should interfere with individuals or corporations only when they place impediments in the road of the exercise of our fundamental civil rights. But there are others who prefer to put the function of government in positive terms. They say that it is the business of the state to create the conditions which best favor the development of individual self-expression. Perhaps it does not matter which of these statements we choose. In the end they may reduce to about the same thing. Thus both may be so interpreted as to lead us to the conclusion that it is the function of the state to guarantee equality of opportunity so that any individual can go as far as his abilities will take him.

It seems to be generally believed that our system of free public education is the most typical expression of this democratic faith that every individual has the right to develop the best that he has in him. But as is readily seen, this immediately brings us around to the conclusion that education has a very positive ethical function to fulfill. This view is now commonly accepted, as the wide-

spread discussion in recent years of the problem of the moral education of youth indicates.

From the statement that education as a whole has an ethical function it follows that logic, as a special phase of our educational program, also has a moral function to serve. We have said that it is one of the important functions of education to teach people to think. But this is also the usual way of summarizing the function of logic, so that the close relation between the aim of logic and the aim of education seems to be generally recognized. We may state this same common purpose in other terms, as follows: The business of education, broadly stated, is to make the world a decent place in which to live. Logic makes its contribution to the realization of this goal by showing men how to secure that intelligent public conscience which is indispensable to the creation of heaven here on earth. It is surely clear that to make the world a fitter place in which to live men must be persuaded to overcome their irrational prejudices, abolish ignorance and superstition, and adjust their differences on the basis of reasonable understandings.

In assigning to logic this very positive ethical function we run the risk of calling down upon our heads the ridicule of those who think it is not the business of teachers (and particularly philosophers) to preach. If the reader will permit a personal reminiscence, reference may here be made to the attitude of one of my own teachers. This very capable and detached philosopher would occasionally declare: "I despise preachers! Great teachers do not descend to trying to convert people. They have no axe to grind." Some sociologists are also taking the same attitude. They are declaring that the social sciences have been stultified by too much ethics, and that if these subjects are ever to become truly scientific they must cast off their moralizing tendencies.

What these individuals overlook is that they are doing the very thing which they condemn others for. It makes no difference whether you preach against sin or against preaching—in either case you are preaching. We are all preachers, more or less. What we need is not *less* preaching, but more of the preaching of the *better* type. But what, then, is *good* preaching? This question is not difficult to answer. We must not attempt to define good preaching in terms of the particular doctrines which are advocated or condemned; rather we must define good preaching in terms of the mental outlook of the person who is doing the preaching. If a man knows his field; if he has reasoned out his beliefs; if he has tried to do justice to opposing points of view; if he tries to avoid dogmatism; if he is willing to admit that in a complex and evolving world no one can lay claim to infallibility; if he grants to other persons the same right to argue their convictions which he claims for himself—if a man has these qualities he ought to be a *good* preacher, and anything he may have to say ought to be worth listening to.

And now, having justified our right to preach, let us return to the more specific matter of how logic can best serve its ethical function in education.

In the preceding chapter we have intimated that the problem of intelligence is to secure a unity of mental life, remove conflicts, and relate the three universes in which we must all live by organizing and coördinating the responses in us which have been built up as a result of functioning in these three worlds. In the present chapter, in developing the subject of how logic can serve the broader interests of life, we must deal with these suggestions in more detail.

We have already stated that these three worlds are not separate and unrelated. There is, for example, a very close relation between the inner world of conscious-

ness and the social world outside. Philosophers of various schools have noted the presence of the social element in much of our thinking. If the reader will but note how his own inner attitudes are made and unmade by social contacts, how we tend to modify our points of view as a result of social intercourse, he will soon discover how intimate the relation of inner consciousness and outer group life is. In the same way, one might proceed to show how the physical world is to a large extent the product of a common social world. Why, for instance, does society put some individuals in institutions for having delusions and hallucinations? Because the world which for them is very real is not socially accessible to the rest of us. We do not share this world with them, and hence we isolate them on the ground that these persons are insane or out of their minds. Thus we see that the test of the reality of the objective physical world is, in the last analysis, a social test. It is obvious, therefore, that the private world of consciousness, the social world of our fellow-creatures, and the common, exterior physical universe are intimately related to each other.

In the psychological literature the term, the "unity of personality," is frequently employed. This is a useful concept. But the self is not only a unitary entity; it is also a complex thing. That is to say, personality or selfhood is both simple and complex. The human individual is really a unity-in-variety: the self may be called a self-differentiating unity or a self-integrating multiplicity. One's personality is the common focus of the three intersecting universes, each of which is itself complex. With respect to our social environment we may say that every individual is a miniature social system. Each time an individual joins a social group he extends his personality. We tend to think of each other in terms of our group

kinships. Every individual is either one of the "in-groups" or the "out-groups"—he belongs to the same club you do or he doesn't. All of us have a different type of reactions, a different self, for different social groups. Usually, of course, there is considerable consistency in our group affiliations and loyalties. We join groups which have ideals more or less compatible with each other. And subjectively the images or the reflections of these groups do not clash in our consciousness.

But if we join groups having ideals which do not harmonize we transfer into ourselves the conflicts which are incipient or real in the groups outside. Of course, if we are sufficiently clever and hypocritical we can "get away" with this fact of membership in incompatible groups. A man who has a separate self for his clergyman, and another separate self which comes forth to meet his bootlegger, may suffer no conflict, so long as he can keep these two selves separated. The situation is apt to become embarrassing only if the man has to present both selves to the outer world at the same time.

Normally there is sufficient consistency in our outer interests and sufficient unity in our inner personality to carry us from one state (self) to another. That is to say, we can *remember* what we did and said in one group, where one self was dominant; this can be recalled by the other self which is uppermost when we are functioning in another group at some later time. There is a very persistent striving to maintain this unity of our inner life, for our happiness is related to the matter of having harmonious selves. But if that principle of unity, whatever it is, is not sufficiently strong to hold these different selves together—perhaps because powerful emotions are driving us in opposite directions—this unity may break up. Then what the psychologists call a "split" personality

develops. The classical instance of this phenomenon is set forth in Robert Louis Stevenson's story of Dr. Jekyll and Mr. Hyde.

We are all multiple personalities to some extent. The main difference between a victim of dissociated personality and the average human being is in the degree of integration. In a split personality continuity of memory fails; there is amnesia, so that what one does as one self is not carried over into the other self which may replace it. Persons who are thus afflicted are not to be envied.[1]

This unfortunate state of affairs can be prevented if we are able to control our impulses and desires sufficiently so that there is congruity in our response systems. The problem of living a life of inner harmony is the problem of how to live a reasonable life. An intelligent existence is one which is planned well enough so that one retains his normal mental balance. A healthy mind is one which has preserved its elasticity, its power to organize and unify. Several qualities are essential to what we may call the normal or healthy mind. These are:

(A) A sense of humor.
(B) Not too many inhibitions or repressions. Too many unsatisfied emotional cravings are dangerous.
(C) But enough restraints to avoid running amuck of society.
(D) A broad tolerance. One of the best antitoxins tending to prevent fanaticism is a knowledge of the history of human thought, a sense of perspective, and a mild state of disillusionment.

And now, having eulogized the balanced, healthy mind, we must proceed to qualify, perhaps even retract, some of the things we have said. It is a paradoxical thing that the very things we set out to accomplish are the things we do not want to accomplish too thoroughly. For ex-

[1] Cases of split personality are discussed by Morton Prince in his book, *The Unconscious,* and by Dr. H. H. Goddard in his volume, *Two Souls in One Body?*

ample, the goal of our concerted social efforts is the production of a perfect society. And yet if we achieve a perfect society there might not be anything more for us to do. Like Alexander, we would sit down and weep for more worlds to conquer. Again, one of the aims of religion is to overcome evil, put Satan on the run, and institute the triumph of righteousness here on earth. But if the clergy accomplish this too thoroughly they will have nothing more to live for. The necessity for their existence might seem to be removed.

And in an analogous way, the statement that we should so order our lives that we are not torn apart by conflicts is true only up to a certain point. Educators apparently believe that inferiority complexes are bad things to have, and therefore ought to be prevented from developing in children. But one writer who has considered the effects of this under the title of the "golden complex" argues that it is the presence of this complex in some individuals which has led to the exceptional creative activity displayed by some of the geniuses of the race. Lord Byron, for example, was sensitive about his lameness, and by way of compensation for this defect—that is, to establish his superiority—he used up the energy, which was made available by his emotional sensitiveness, in writing poetry. Napoleon was a Corsican. His sensitiveness to the fact that Frenchmen considered Corsicans inferior to themselves led him to exert himself so that Frenchmen would recognize his worth. And so we might add to the list.

From this point of view it seems that conflicts, if not inhibitions and repressions, may serve as catalytic agents which activate the mental processes so that an individual becomes capable of unusual efforts. This, however, does not contradict anything we have said. No matter what the source of a man's creativity may be, he must work

within certain limits. These limits are set by human nature and by society, working together. A man whose responses are so unconventional, so extreme, that others consider him erratic, eccentric, or even crazy, has little chance of finding that social approval which human beings normally desire of one group or another. And it is equally true that an individual whose inner life is a constant civil war can never get anything done.

Perhaps it is fortunate that some individuals have been the victims of different types of abnormalities of psychic life. A really original idea calls for a departure from traditional and conventional views. Sometimes it happens that the greater the idea the crazier it appears to a man's contemporaries. We have already noted this fact that the driving force behind a new scientific theory resembles in its mechanism the obsessions of insane people. This is pointed out in one way by William James:[2] "When we speak disparagingly of 'feverish fancies,' surely the fever-process as such is not the ground of our disesteem—for aught we know to the contrary, 103° or 104° Fahrenheit might be a much more favorable temperature for truths to germinate and sprout in than the ordinary blood-heat of 97° or 98°." As various writers on genius and insanity have pointed out, great geniuses have sometimes been considered insane in their own age, but it is because later generations have come to accept their ideas that they cease to be considered insane and become geniuses who were in their own age ahead of the times. The history of creative insanity still remains to be written.

As already stated, from the point of view of the individual who is thus afflicted this is an unfortunate frame of mind to be in. But society in the long run may occasionally profit by the presence of such persons. The great

[2] *The Varieties of Religious Experience,* Ch. I.

majority of crazy people in this world are the victims of what, from the point of view of society, is unprofitable lunacy. But once in a while a real genius appears from among them. Just as society profited by the fact that Beethoven lost his hearing, and was thus driven back upon himself to write the musical compositions which gave expression to the resulting emotions accompanying this personal tragedy, so society may benefit as a result of the complexes and other psychic disturbances of mental equilibrium which shake the foundations of personality. But let us not forget that the resulting products are an attempt on the part of the individual to reëstablish a unity and harmony of inner life. It still remains true, therefore, that we must achieve enough of a dynamic equilibrium to enable the separate selves of our partially integrated personality to coöperate in some degree.

Society need not fear that the time is coming when we will all be so well adjusted to ourselves and to others that the creative effort produced by the conflicts with groups without and of selves within will be lacking. The fact is that as we solve old problems, make readjustments between conflicting group interests, and better the old environments, we inevitably produce new conditions which present novel problems. Life is activity; the only goal of life appears to be further activity, more growth, so that as individuals and social orders go on there is always sufficient incentive to challenge the best there is in us. Indeed, it almost seems as if we are creating problems more rapidly than we are capable of solving them, and that in the future an even greater drain will be made upon the potentialities of human nature.

There is no reason to fear that we will ever produce the ideal society, if by that term we mean a society in which all problems are solved, where all individuals are perfectly adjusted to their physical and social environ-

ments, and hence where there is nothing more to be done. Heaven on earth is not a fools' paradise; neither is it a nirvana of inactivity: *it is a society in which each individual has enough to do of the sort of thing he likes to do and is capable of doing to make life interesting.* Whether such a heaven must always remain the vision of unfulfilled desire of utopian dreamers is but another of our many problems.

Part II

LOGIC AND THE SCIENCES

Chapter I

WHAT IS SCIENCE?

THE terms science, religion, philosophy, and art are common terms. Their frequent use indicates presumably that each of these terms symbolizes some type of activity or interest quite different from that which is designated by the other terms. This must be the case if we are to speak, for example, of a "conflict of science and religion." We also hear of a hostility between science and poetry. Again, the question is occasionally raised of whether art has any kind of moral function to fulfill. Such oppositions as these indicate that these terms must stand for some difference in interest or in the kind of activity involved.

And yet it is not easy to discover just what it is that distinguishes one of these fields from another. Just where does science leave off and religion begin? Where can you draw the line between art and philosophy? Further reflection may well lead one to suspect that, strictly speaking, there is no such thing as *Science,* but only individual scientists. The same might be said for any of these other pursuits. We sometimes hear the expression: "Modern science says thus and so"—whatever the claim may be. But the modern nominalist—the person who says that abstractions like justice, humanity, science, and the like are mere *names*—might argue that there is no such thing as "modern science," but only the opinions of individual men who call themselves scientists.

Yet the fact that certain individuals are classed together and called scientists, religionists, artists, and philosophers, indicates that there are classes of activities

which can be distinguished from each other. And now, having decided that science, in some sense or other, exists, the next question we must consider is this: What are the characteristics of science? What are the essential qualities and attributes of the scientific attitude and the scientific method?

Here we find a number of answers presented. Some thinkers would restrict the meaning of the term science to a relatively narrow field of investigation. Those who adhere to this interpretation of science would put certain types of problems within the field of science and exclude a consideration of other kinds of problems, which might then be considered as falling within the field of metaphysics, religion, or art. Herbert Spencer, for example, considered that religion was concerned with what he called the Unknowable, whereas science was concerned with the Knowable. Others would enlarge the domain of science and include within its scope the entire natural universe, at the same time denying that there is anything left over which could fall within the province of metaphysics or religion.

In his work, *The Grammar of Science,* Karl Pearson, the eminent statistician, takes this stand. He tells us (Chapter I) that the field of science is unlimited; its material is endless; every group of natural phenomena, every phase of social life, every stage of temporal development, is material for science. "*The unity of all science consists alone in its method, but not in its material.* . . . The material of science is co-extensive with the whole physical universe, not only that universe as it now exists, but with its past history and the past history of all life therein." The goal of science, Professor Pearson tells us, is nothing short of the complete interpretation of the universe.

The general opinion among scientists seems to be that

the field of science is limited to phenomena to which a certain type of analysis and method of investigation can be applied. The belief is that science is concerned with facts which can be observed through the senses or inferred from such observations. Whether there is any other type of reality and knowledge is a philosophical problem which science does not presume to investigate. If this view of the nature of science is correct we may formulate the following definition of science: A science is a body of organized principles, derived from the observation of facts, and acceptable to the experts in that field. This definition is not intended to be rigorous and final. It fails to enunciate clearly the several necessary tests of a science. Unfortunately, the experts are not yet finally agreed on this matter. Several tests of science have been proposed. Science may be defined either (*a*) in terms of the *method of procedure,* or (*b*) in terms of its *content.* According to the first test, advocated by Karl Pearson, the test of whether a subject is scientific is merely the problem of determining whether the study employs scientific method. The second way of determining whether a given study is scientific is to look at the content—can one discover or formulate laws governing the behavior of the entities in this field, and is there any hope of explaining such laws as are discovered?

It is the failure to distinguish between these two tests which is responsible for many disputes. The question is sometimes argued as to whether the more recently formulated and less exact sciences, such as economics, sociology, political science, and psychology, are really sciences. For instance, there are those who hold that sociology can never hope to be a real science. The question is also sometimes raised as to whether ethics can be called a science. These questions are readily answered if we specify what our test of a science is. If our test of a

science is the ability to vary at will the factors responsible for a phenomenon, observe the corresponding variations in the result, and formulate laws so that in the future one can predict what will happen under certain specified conditions, then obviously ethics and sociology are hardly to be called sciences. There is very little power of experimental control and prediction in social phenomena; the social sciences are not nearly so exact as the physical sciences.

But if you mean by a science any study which investigates its phenomena by *scientific method,* then there is no reason why the social sciences, including ethics, should not be considered as scientific as physics. Scientific method has become a fetish with modern thinkers. The reason for this is that it is supposed to be responsible for the truly remarkable advances which the sciences have made. The chief characteristics of scientific method are careful observation, classification of the facts observed, the formulation of laws of relationship between the facts observed, always testing these generalizations by reference back to the facts, and always excluding one's own prejudices in the processes. The term, the "objectivity of science," eventually reduces to the fact that the scientist wants to know *what is* rather than *what ought to be.*

Judged by either test, logic is as much of a science as any other discipline. Indeed, if logic is not a science then how can any other field of knowledge, each of which necessarily employs logical processes, claim to be a science? Logic satisfies both of the tests we have set forth. On the one hand, logic has its laws, the laws of thought, which, however, are not merely laws of thought but also laws of reality.[1] On the other hand, it is clear that logic also employs scientific method. A perverse critic might

[1] These laws are stated in the following chapter.

raise the question of whether logic has any "facts" in the observation of which scientific method can be employed. Replying to this we note that while it is true that we admit that logic deals more with the forms of reason than with the substance of reasoning in a particular field, nevertheless these forms are abstractions from concrete cases of reasoning applied to empirical facts. To clinch the argument we need only point out that, after all, scientific method is simply logical method.

In its final and perfect state every science would assume an abstract form which would be the same for all science. The following pattern, while laying no claim to perfection, has the advantage of summarizing the mathematician's method of procedure. Various authors, including Kant, have noted that as a science progresses it tends to model itself more and more after the pattern of mathematics, which is approximately as follows:

MATHEMATICS

CONCEPTS USED— a. Point b. Line c. Surface, etc., etc.
POSTULATES— a. The whole is equal to the sum of its parts b. Things equal to the same thing are equal to each other c. Etc., etc.
THEOREMS— a. The theorem of parallels b. The Pythagorean theorem c. Etc., etc.

This, we have said, represents the ideal pattern. A perfect science would be one in which one could so define

the fundamental concepts and so frame the postulates that all the remainder of the phenomena in that field could be deduced from a small set of such concepts and postulates. Unfortunately—or perhaps fortunately—nature as a whole cannot be reduced to an infinite deductive system. There are new things in nature. At the new and higher levels old laws are superseded by new laws. Hence each science, while it is built upon its predecessors (as biology is based on chemistry and chemistry on physics), must nevertheless formulate such additional laws as will make it possible to describe these novel uniformities of sequence in this new field. The postulates, however, in every case are those simple and universal principles (in this field) whose validity (in every case except perhaps in mathematics and logic) have been established by experiment, and from which the more complicated phenomena in that field can be deduced.

This procedure forces the scientist to attempt to formulate fundamental principles upon which all the experts in his field can agree. Just as in Euclidian geometry there is a common body of definitions and postulates (often called "axioms") which all will accept when they are thinking in that field of mathematics, so in physics there are similar postulates—such as that the same body cannot be in two different places at the same time. It seems obvious that such sciences as sociology and ethics, where disputes and misunderstandings are so common, can hope to make progress only when the men in these fields have agreed to follow the mathematician's method of procedure. How can we ever hope to agree about the more complex phenomena if we cannot agree about a few basic definitions and assumptions?

Chapter II

WHAT IS LOGIC?

STUDENTS working in the special fields of human knowledge usually feel called upon to define the nature and the limits of their particular subjects. This appears to be an admirable practise and there is no reason why we should not follow the precedent. Let us therefore consider the question: What is logic?

One possible way of defining logic is to point out that it is a cross between psychology and mathematics. But this, for several reasons, does not appear to provide a very promising beginning—especially if we have in mind the following definitions of these two subjects. Some of my readers are undoubtedly familiar with Bertrand Russell's famous definition of mathematics as the science in which you never know what you are talking about or whether what you are saying is true. And psychology, as one wag has wittily said, is the subject in which you talk about things which everyone knows about in terms which no one understands. Combining these two to form the definition of that hybrid subject, logic, gives us something resembling the following: Logic is the science in which you study more and more about less and less, until you know everything about nothing! Whether this formulation follows from the antecedent definitions or not, as a statement of the nature of logic it is quite satisfactory. Logic is the most general and most abstract science in existence. It has little empirical *content,* being concerned chiefly with the *forms* of reasoning in general. The recent tendency to identify logic with the *science of order*

or the *analysis of systems* is but another way of saying the same thing.

The essential thing about reasoning is *inference*. Inference is the process by means of which we pass from whatever is given (as self-evident or as already demonstrated) to something which is believed to follow necessarily or probably from that which is already accepted. Thus the passage from the presence of clouds to the coming rain which those clouds *signify* or *mean* is a simple case of inference. Since, in general, the modes of procedure are fairly well conventionalized, we may say that *logic is the science of the methods of inference*.

Man is a logical animal because he is a rational animal. He is a rational animal because he knows how to use *tools*. Indeed, one writer proposed to call man *homo faber*. A tool is an instrument which is used as a *means* to achieve an *end*. Language, which is the chief instrument of thought, is also a tool. In the days of their origin language and thought were both tools, means employed towards the end of biological survival. But now, fortunately, thought is occasionally detached from its utilitarian purposes; it has become an end in itself, satisfying for its own sake. The same may be said for language when it is serving some esthetic purpose, as in poetry or drama.

There is a very close relation between *inference* and *knowledge*. If the results which we infer turn out to be true we say that we have knowledge. Knowledge is not only the means by which man *adapts* himself to his environment, but it is also the tool by the effective use of which man *re-creates* the environment to suit the new needs and ends which arise in the life-processes. In this process of give-and-take between man and his various environments man's very nature is transformed, so that life is carried to new, if not higher, levels.

The term knowledge is of course still vague. For present purposes, however, it will suffice to point out that there are two kinds of knowledge:

Self-knowledge
Knowledge about the external world

It has been argued by some authorities that we have made greater strides in securing knowledge about nature than we have about human nature. In view of the intimate connection between the two this may be questioned.

Our lack of knowledge about ourselves, that is, about the nature and *modus operandi* of the human mind, is manifested by the fact that we cannot be sure whether the present ways or methods of inference are ultimate and absolute, or whether they will some day be superseded by other modes of inference. Bergson and others have argued that in addition to reason there is another instrument of understanding, of arriving at knowledge, which is what is called intuition, a "feeling with" the thing to be understood. But in reply to any doctrine which asserts that reason has its limits we may point out that if there is a method of arriving at knowledge through intuition, and if there is some sort of lawfulness in the process, then this method of inference is also subject matter for logical investigation. This, however, is only another way of repeating our statement that logic is the most fundamental science in existence.

The statement that logic is more fundamental than any other science (even including mathematics) may be fortified by a few examples of the assumptions which science makes which it is the business of logic to examine.

(A) In the first place, every science, since it employs reason, takes over the laws and principles of logic. We are not questioning the necessity or the utility of this, but inasmuch as the whole question of the ultimate logical

sanction for this procedure still remains to be established, logic can claim a more fundamental rôle than the sciences which use logic. Since we will have numerous occasions to use these laws of thought we will set down the following three laws, which are common to both branches of logical inference, deductive and inductive; later on we will add to them the other assumptions which we will feel the need for in the empirical (experimental) sciences:

1. *The Law of Identity.*—A is A; whatever is, is; a thing is what it is.
2. *The Law of Contradiction.*—A is not not-A; a thing is not what it is not.
3. *The Law of Excluded Middle.*—A is B or A is not B; any term is included in a given class or it is not included in that class.

(B) In the second place, science takes for granted the reliability of sense perception. That is to say, it implicitly accepts a postulate covering the assumed existence of a world of facts (objects) which are independent of the mind or the knowing organism.

(C) In the third place, science assumes some sort of postulate covering the correspondence between what we call "ideas" (such as the number system, and other intellectual constructs) and the objective realities of the external world. This is illustrated by the fact that while mathematics is employed as much as possible in all the sciences, we get into intricate problems when we try to discover the relation, for example, between the conceptual space (or spaces) of mathematics and the so-called real space which physics is supposed to be concerned with.

The problems of time and space, of the relation of physiological, conceptual, and physical realms and realities have come more to the front since the appearance of Einstein's theory of relativity. Many individuals have

noted that in recent years science, and particularly physics, has become more metaphysical, which is but another way of saying that scientists are becoming more conscious of the necessity for investigating the logical, epistemological, and metaphysical presuppositions which form the foundations of science.

Summing up the foregoing points we may say that logic, as the foundational science of the methods of inference, is a study of the ultimate laws or postulates which enable human beings to achieve knowledge. This knowledge is usually made accessible through concatenations of words, so that language appears as the conventional tool which the mind employs in the organization and transmission of the results of its exploratory processes.

Chapter III

LOGIC AS A NORMATIVE SCIENCE

THUS far we have set forth the reasons for believing that logic is to be counted among the sciences. But this cannot be the whole story. If it were it would be hard to see why, and in what way, the treatment which reasoning receives at the hands of the logicians differs from the treatment it receives at the hands of the psychologists. And yet there is some difference, as a perusal of the treatment of this subject in the text books in these respective fields will demonstrate.

The outstanding difference can be summed up in the statement that while both logic and psychology are interested in mental processes (including reason), the attitude of the two towards the phenomena of mental life is different. Both logic and psychology examine perception, conception, judgment, and reasoning; the difference in interest consists in the fact that psychology is interested primarily in the analysis and description of conscious states (so-called), while logic is interested in the *value* of the products of such states and processes. Logic is interested in the truth or falsity of ideas, propositions, and beliefs. The situation may be summed up in the following way:

Descriptive Science—Facts
(What is)

Normative Science—Values
(What ought to be)

The normative sciences are the following:

Normative Sciences { Ethics, Logic, Esthetics }

The necessity for this distinction is established by the fact that human beings are not content with things as they are, but are interested in things as they might be or ought to be. This attitude, where facts and performances are evaluated as good or bad, manifests itself particularly in those fields where facts are plastic and capable of being altered. This is notably true in the field of human activities. Hence the normative sciences are concerned with the setting up of standards of perfection by means of which the actual achievements of men may be measured as being worthy of emulation or deserving of condemnation.

But, someone objects, is it not true that psychology is also concerned with "norms"? Are there not standards for performances in the field of mental tests? Do not those who deal with delinquent children evaluate their defective behavior and attempt to correct their shortcomings? Replying to these questions, we must admit that the psychologist does talk about the norms for his mental tests. But we must notice that the "norms" of a psychologist are not to be confused with the norms of a normative science. When a psychologist "standardizes" a test he gives the test to a large, and preferably unselected, group of individuals of a certain age—let us say he gives the test to the pupils in the third grade of school, most of whom are about nine years old. He then finds out what the average child of nine years is capable of doing. The performance which the grade represents is defined as the *normal.* It is, you see, a *statistical average* of what the normal (average) child is capable of doing. If one child

does better than the majority he is said to be supernormal, while another child who does poorer than the average is said to be subnormal.

We must now point out that the norms of thinking which the logician sets up are not statistical averages. The ideal (norm) of thinking in logic is not the thinking of the average man, which may provide anything but a satisfactory standard. The ideal standard which the logician holds up may not be achieved by anyone at the present time, whereas the standard for a mental test is achieved by the majority of individuals. The ideal standards of a normative science (in logic, perfect consistency; in ethics, perfect behavior; in esthetics, ideal beauty) may go beyond experience; they may be "ideals" in the sense that they give us something to work towards, even though we cannot as yet measure up to the standards. To be sure, our ideas of what ought to be are related to what is; our ideals come out of experience, and yet they go beyond experience. This is the paradox of every normative science.

Moreover, we must also note that so long as the psychologist who studies thinking is a scientist (of the descriptive sort) he remains ethically neutral. He studies the facts of human behavior and experience without ethically evaluating them. This is clear when a psychiatrist is investigating insanity. The psychologist asks, *What are the facts?* He is not interested in the question of whether they are good facts or bad facts. *As a scientist* it is not his business to remove or better subnormal or pathological conditions.

It is true, however, that most psychologists are interested in helping society. They do give advice. Clinics are established to aid human beings. But this does not contradict our statement that the psychologist (as a descriptive scientist) is not interested in an ethical evalua-

tion of facts. The way out of the apparent contradiction lies in pointing out that every scientist is something more than a scientist—he is a human being. When a student goes into the laboratory he adopts a characteristic mental set—this is the frame of mind he is in when he is proceeding according to the canons of scientific method. In that frame of mind he excludes his own sentiments, his own set of values. But in addition to being a scientist he is a citizen of the state—perhaps also a husband and father—and as such he has social ties. The scientist has the same social obligation to help his fellow creatures which anyone else has. Hence the psychologist as a human being has the right and the duty to indicate remedial means after he has made his diagnosis. But when he evaluates facts according to some ideal the descriptive scientist ceases to be the abstract thinking machine he was and becomes a human being, a philosopher in fact. He passes over into the field of the normative sciences. If the scientist is aware of the fact that what he now says is being advocated as a matter of personal belief, involving a scale of values, there is no cause for objection. The danger results only when a man philosophizes, while still thinking that he is speaking as a scientist. Failure to make this distinction between the function of the natural sciences and that of the normative sciences has made it possible for some dogmatists to denounce the obscurity of philosophy while yet themselves becoming philosophers without apparently realizing it.

It is because logic and ethics not only have the problems of the descriptive sciences on their hands, but also the problems of the normative sciences, that the conclusions in these fields are so much less certain. The normative sciences are as good sciences as any so long as they are concerned with the observation of facts (mainly the gathering and arrangement of historical materials), but

when they, in addition, try to draw from these facts ideals for the future guidance of mankind they lay themselves open to difficulties and defects which the descriptive sciences do not have to contend with. For valuation introduces an emotional element which is not present in the fact-finding interests of descriptive science. A value is a fact, or a desired fact, to which we attach some affective coloring.

This attitude of approval or disapproval is as hard to exclude from logic as it is from ethics. When one surveys an argument one naturally falls into the frame of mind of a person who is viewing a work of art or passing judgment upon the conduct of an immoral person: you are inclined to say, "I like it," or "This is bad," etc. *Judgment,* which is the ultimate psychological basis of reasoning, almost inevitably includes this evaluating attitude of the self. Judgment is the interpretation which is added to the facts. Reasoning occurs when we judge that something is true because we hold that something else is true. Whatever is disputable stands in need of verification, and in logic it is hard to draw the line between logical and ethical validity. Here we refer to our previous statement that good conduct is based upon intellectually honest thinking and good logic is ethically desirable. Truth is a value as well as an occasional fact.

The absolute inescapability of the psychologist's transition from the norms of his descriptive science to the norms of a normative science is manifested in the fact that when a psychologist attempts to think straight in his own field, when he tries to formulate a system of principles for his own science, he thinks as logically as he can. That is to say, science aims at truth, and scientists try to think, not according to the standards of the average man, but according to the ideal standards of a perfect science.

If you should ask a scientist why he strives for accuracy, why he desires consistency, why, in short, "scientific method" is his pattern, he will eventually be forced to justify this belief and practice in terms of his obligation to his fellow scientists or to society as a whole. If a scientist has reasoned out the *rationale* of his chosen profession the ethical element will undoubtedly enter somewhere—presumably as a result of the respect he has for intellectual honesty. The vast mass of facts in different fields which now stands as a monument to the success of science is also testimony to the noble desire to know the truth. Progress in the winning of truth is inherently conducive to progress in ethics. Socrates was right when he stated that there is a fundamental interdependence between intelligence and morality.

Chapter IV

LOGIC AND RHETORIC

DR. JOHN B. WATSON, well-known exponent of the behavioristic movement in psychology, has told us that thinking is merely talking to one's self; it is the implicit movement going on in the vocal organs. This theory has provoked much debate. There is a doctrine that those who talk most think least, but if Dr. Watson is correct there are good reasons to doubt the truth of the doctrine. Those who talk most think most, according to behaviorism. But perhaps this is not entirely fair to this "every little movement has a meaning of its own" theory. Perhaps there is some peculiar virtue about "implicit" movements of the larynx, so that those who talk to themselves talk more wisely than those who talk aloud. Let us hope so! In that case it would still remain true that while speech is silver, silence is golden. We could then continue to believe that "still water runs deep"—as deep, in fact, as our wind-pipes!

Speaking seriously, however, we may note that there is indeed a very close relation between thinking and the use of language. While it is true that now a person may think without the use of verbal language—he may, for instance, think in terms of mathematical symbols or musical notation—it is doubtful whether men could ever have developed to that point without language. Animals do not, so far as we know, employ articulate language, nor do they think in abstract terms. Perhaps, therefore, the two are inseparable.

A word is a name we give to an object or a process,

and once having words we can think in terms of these written and spoken symbols without pointing directly to the entities which they are supposed to symbolize. That is to say, words become substitutes for objects, making it possible to live in a world of abstractions. Thus we come to live in a universe of words and a world of objects.

Various writers have pointed out that the most useful things in life are frequently the most potent for mischief, if they are not properly used. Drugs, dynamite, radios, and all other inventions of man can be as great curses as they are boons. The same is true of language. Men may do much good and much harm through the use of this invention. There is a saying that the pen is mightier than the sword. This has been true to some extent in the past, and it is becoming truer as time goes on. But there is no guarantee that either the sword, pen, or the tongue will be used only in the interest of justice and goodness.

The explanation of the power of the written and spoken word is to be found in the fact that language serves several functions; its appeal to man is through several channels. One of the most valuable distinctions which C. K. Ogden and I. A. Richards have introduced in their book, *The Meaning of Meaning,* is their differentiation of the several functions of language. These are as follows:

The two uses of language { *Emotive use*; *Symbolic use*

Now logic is not the only subject which studies the structure of language, its relation to thought, and its effective use in speech. Such subjects as rhetoric, grammar, and dialectic, are also concerned with these matters. These subjects, however, are usually thought of as con-

stituting an applied art, the art of persuasion. The effectiveness of rhetoric lies in its power of convincing people. And its effectiveness in the hands of public speakers does not rest on the symbolic use of language, but on its emotive use.

It is obvious that if we are to escape the intellectual debauches which accompany the sentimental explosions released by the emotive use of language rhetoric must be made subordinate to the purposes of logic. Plato and Socrates developed a hearty contempt for the Sophists who, taking advantage of their technical skill as debaters, sought to make the worse appear to be the better reason. The modern representatives of the Sophists are no more interested in arriving at the truth than were their predecessors; their primary concern is to win the argument and get the decision. Our contemporary salesman, who is so essential to "big business," has a "smooth line," and can "deliver the goods." The terminology and the technique of salesmanship is insinuating itself so thoroughly into our habits of thought that some are beginning to fear that it is corrupting our intellectual integrity. Teachers now frequently speak of culture as though it were a meal to be sold in the school cafeteria. Indeed, much of the present volume, I fear, must appear as a talk to break down "sales resistance" against logic. Well may we, like Socrates and Plato, begin to fear those silver-tongued orators who sway the public by their appeal to sentiment.

No one denies that the study of grammar and rhetoric is essential to an adequate expression of ideas. But grandiloquent phrases must not be sold for clear logic. A saccharine style is the enemy of meticulous accuracy. For ordinary purposes we use words loosely and indiscriminately. We think we understand each other, but as a matter of fact language, in most cases, does not convey ideas, but is a means whereby we scratch each

other's back, so to speak. In exact thinking, in science and philosophy, carefully defined terms are essential. In economics, for example, the terms wealth, capital, money, etc., are technical terms, carefully delimited in their connotation. But in popular thought they are used interchangeably. Probably no particular damage is done here, but in other cases (we shall have occasion to examine some of these later) the damage done is considerable, especially since we are unconscious of it.

It is for this reason that logic is recommended as an indispensable antidote to the insidious art of persuasion. While we cannot get rid of emotion, we can at least restrain it to those occasions where (aside from recreational activities) it can be directed by intelligence. In demanding careful and exact thinking about its own subject matter logic tends to produce carefulness in the study of all subjects. As one writer has put it, logic familiarizes us with the task of examining our own reasoning to see whether our conclusions are conclusive. It helps us to understand the conditions of proof and verification. It develops in us the habit of criticism, of weighing propositions, of examining evidence. It gives us the satisfaction of knowing that when we believe a proposition is true we take this stand because we are not merely following the crowd. With Aristotle you may then say, "That which other men do under compulsion I do freely,"—because you then have a reason for doing it. The only freedom in this world is the freedom which reason gives, as Spinoza has shown us.

This exaltation of reasoning may perhaps meet with the criticism that too much thinking is a bad thing. There is a common belief that some men have become inhuman, intellectual monstrosities even, as a result of their scientific pursuits. Probably the atrophy of Darwin's interest in music as a result of his absorption in science will here

be cited. But as a matter of fact the trouble with such individuals is not that they have thought too much, but that they have not been well balanced in their choice of subjects to think about. There is a vast difference between thinking too much on the whole and thinking too much about some one subject. The former can hardly occur. Aristotle was probably right when he pointed out that while most virtues are a mean between extremes, in the case of reason this formula does not work. But one can develop a single-track mind by thinking too much about some one thing. However, as we have already said, the price which genius sometimes has to pay for high proficiency in one field is the underdevelopment of interest and ability in other fields. But this need not be the case. One can attain a well-rounded development if one so orders his life as to secure balance and proportion.

In any event, there is no justification for letting the magicians of language put our critical faculties to sleep by giving us intellectual opiates, with the intention of then stealing away from us that which it is their purpose to gain. Let us therefore inoculate our minds against those germs of mental sleeping sickness which are injected into us by the verbal hypodermics of those who, for their own nefarious purposes, would render us critically unconscious.

Chapter V

LOGIC AND MATHEMATICS

IN the past logicians have tended to assume that all the principles by means of which thought arrives at valid conclusions have already been formulated. But it is now generally recognized that the logical principles governing inference are still in process of formulation. Therefore, any definition of logic which suggests that it is static and complete is premature. One of the best illustrations of the fact that logic is still in its developmental stages is to be found in the recent attempts to unify mathematics and logic in the domain of mathematical logic.

Perhaps before we proceed to discuss this problem of the relation of mathematics to logic we should, if possible, get clearly in mind what the relation of pure mathematics to natural science may be. We say "if possible" advisedly, for if you go to the mathematicians for a definition of their subject it will be found that they are not entirely in accord on the matter. The problem of defining mathematics turns out to be a more difficult task than it at first sight appeared to be.[1]

But no matter how divergent the various definitions may be, mathematicians are pretty well agreed upon at least two essential characteristics of mathematics: in the first place, that mathematics is concerned with inference, and, secondly, that mathematics is abstract, so that its propositions hold for anything whatsoever which satisfies the conditions imposed by the domain of mathematics

[1] This has been pointed out by Robert E. Moritz in his paper on "Meaning, Methods and Mission of Modern Mathematics," in the *Scientific Monthly*, May, 1928.

wherein this proposition, or theorem, is accepted. This statement, which no doubt appears obscure enough, may be illustrated by referring to a simple case taken from the realm of number.

The development of mathematics from the most primitive systems of enumeration, such as the practise of tying knots, or cutting notches, or piling up sticks or stones, to the modern theory of number as the "class of all similar classes" involves the long and tedious intellectual journey of the race. To get to our present point of vantage the transition had to be made from the savage, whose earliest distinction was between "one" and "many," to the idea of addition as a process of increasing in succession single units to form a group. The separation of number as an abstraction from number as identified with concrete objects and the further step of representing a number by a symbol were also essential if mathematics was to achieve that abstractness and generality which are its present power.[2]

According to Bertrand Russell, mathematics may be defined as follows: "Pure mathematics is the class of all propositions of the form '*p* implies *q*,' where *p* and *q* are propositions containing one or more variables, the same in the two propositions, and neither *p* nor *q* contains any constants except logical constants." [3]

For the layman this is probably not a very enlightening definition. But the statement "*p* implies *q*" gives us sufficient information to allow us to pass on to an essential difference between mathematics and physics. This is summed up by one writer [4] in the following words: "The

[2] On this matter of the abstractness of mathematics consult Dr. A. N. Whitehead's two books, *Science and the Modern World* (p. 32), and *Introduction to Mathematics* (p. 15), the latter volume appearing in the Home University Library.

[3] *Principles of Mathematics,* p. 3.

[4] A. W. Stern, "The Rôle of Mathematics in Modern Physical Theory," in *The Monist,* Vol. 39 (1929), pp. 262-273.

difference between a mathematical proposition and a physical proposition is not one of form; both are of the nature '*p* implies *q*.' The difference is that in mathematics the properties of '*p*' are given by definition, while in physics the properties of '*p*' are discovered by observation." This distinction between the two methods of formulating definitions is valuable, and we will have occasion to make use of it later on. At the present time all we need note is that the mathematician possesses a certain logical freedom: he is not constrained by physical reality, which limits the freedom of choice of the physicist so that what he says must "fit the facts."

But logic, we have previously seen, is also concerned with inference. How, then, does the logician's interest in inference differ from the mathematician's? The process of inference, which when validly carried through gives us consistency, has been summed up in various forms, but one of the most cryptic (though really very adequate—once you understand it!) is that which says: "If not-p implies p, then p is true." This statement [5] only means that "a proposition which follows from the hypothesis of its own falsehood is true."

This way of stating the essence of inference makes no differentiation between logical and mathematical inference. But this failure to discriminate between the domains of logic and mathematics would not be regarded as a shortcoming by Mr. Russell, for it is his contention that mathematics and logic are one. His view is summed up in the following enthusiastic declaration: "The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age. . . ." [6]

This conception of the relation of mathematics and logic follows directly from the definition of mathematical

[5] Quoted from Whitehead and Russell, *Principia Mathematica,* Vol. I, p. 108.

[6] *Principles of Mathematics,* p. 5.

logic which Russell [7] gives. "By the name mathematical logic," he tells us, "I denote any logical theory whose object is the analysis and deduction of arithmetic and geometry by means of concepts [such as order] which belong evidently to logic." According to Russell [8] "starting with premises which would be universally admitted to belong to logic, and arriving at results which as obviously belong to mathematics, we find that there is no point at which a sharp line can be drawn with logic to the left and mathematics to the right."

The attempt to get at the fundamental and ultimate elements of thought may be called "logical atomism," adopting a phrase employed by Bertrand Russell. The logical atomist is interested in finding the fewest possible primitive principles, relations, and concepts, necessary to build up all possible forms of thought. Louis Couturat, in his *Algebra of Logic,* enumerates fifteen fundamental principles from which he thinks all forms of logical thought can be built up. C. I. Lewis, in his *Survey of Symbolic Logic,* and Whitehead and Russell, in their *Principia Mathematica,* have also worked out sets of fundamental principles of symbolic logic. There are some notions common to all these systems, but in other cases what is *proved* (i.e., a theorem) in one system is *assumed* in another system, and *vice versa.*

There are other able thinkers who do not accept this identification of logic and mathematics. Henri Poincaré [9] refused to be convinced that "symbolic logic is mathematics, mathematics is symbolic logic, the twain are one" —to use C. J. Keyser's oft-quoted words. Others have also failed to see the identity of the two subjects.

[7] "The Philosophical Importance of Mathematical Logic," *The Monist,* Vol. 23, p. 481.
[8] *Introduction to Mathematical Philosophy,* p. 94.
[9] Cf. *Science and Method.*

H. W. B. Joseph[10] has affirmed that "the identification of logic and mathematics is a blunder against which it is desirable to protest on more grounds than one." Professor Joseph objects to this attempted unification on the ground that while the mathematician thinks and infers, he does not study the activity of thinking and inferring as such. This is a rather subtle point, and the following passage is quoted as an illustration of the extent to which logical refinement can be carried. The writer asks: "Is the nature of thinking which studies the same as the thinking studied? The answer to this question will bear upon the identification of logic with pure mathematics; for if the thinking studied is of the same nature with the thinking that studies it, then the mathematician's thinking, which it belongs to logic to study, will be of the same nature with the logician's thinking." For this and other reasons, Professor Joseph is unwilling to accept this identification of mathematics and logic. Somewhat the same attitude is taken by Professor Whitehead[11] when he states: "The justification of the rules of inference in any branch of mathematics is not properly a part of mathematics; it is the business of experience or philosophy. The business of mathematics is simply to follow the rule."

This dispute may appear very academic and unprofitable. But as a matter of fact, a final decision about this matter would be of fundamental importance, for on this issue hinges the question of which science, mathematics, logic, or psychology, is to have the last word. Our own view, set forth in the statement that logic is the most fundamental science in existence, definitely commits us to the doctrine that, in the last analysis, logic alone

[10] "Logic and Mathematics," *Journal of Philosophical Studies,* Vol. 9 (1928), pp. 3-14.
[11] *A Treatise of Universal Algebra,* p. vi.

can give us the proper justification (if such there be) for the laws of thought and the principles of inference.

This completes our summary of logical atomism or mathematical logic, the point of view first envisaged by that universal genius, Gottfried Leibniz.[12] The value of such an analysis of thought into its simple logical elements is that it exhibits the formal processes by means of which we, psychologically, make the inferences which consist in relating new experiences to old knowledge. These are the processes which we all use in thinking. Any account of thinking, whether by a Behaviorist or an Intellectualist, which ignores these processes or fails to explain them stands condemned at the outset, for it is through the use of these logical operations that any system of science can hope to establish itself. If Behaviorism, for example, is to demonstrate itself to be the "true" system of psychology it must show how logical categories are deducible from its own categories of *stimulus, response, conditioned reflex,* and the like.

[12] Leibniz's place in the history of Logic is discussed more fully in Chapter VII.

Chapter VI

LOGIC AND PSYCHOLOGY

THE intimacy and the character of the relations which obtain between logic and psychology depend to a large extent on the type of logic and the type of psychology which are being considered. At the present time there are a number of different schools of psychology. William James summed up the situation very well when he observed that psychology today is but the hope of a science. Perhaps the best way to come to an understanding of the existing situation in psychology is to survey briefly the development of this subject in modern times.

Modern psychology, like modern philosophy, has been much influenced by the views of Descartes, who deserves to be regarded as one of the founders of modern thought. Descartes' doctrine in the field of psychology is known as the *interaction* theory of the relation of mind to body. According to this view, there are two distinct realities responsible for human nature as we know it: soul (or mind) and body. Descartes' explanation of how these two interact was not very satisfactory and we will not consider this phase of his views. We are more concerned with the theory of knowledge—the epistemological theory—which Descartes advocated as a part of his general psychological doctrine.

The theory of knowledge which Descartes accepted is called *rationalism.* The rationalist asserts that to get knowledge we must use our reason. This doctrine is to be contrasted with the doctrine of *empiricism,* which

states that to gain knowledge we must use our senses. The empiricist tells us that our ideas (or concepts) are derived from percepts. The rationalist holds that our knowledge does not come through our senses; it is innate, he says, born within us. Hence to get knowledge we must look within ourselves and call upon this intuitive storehouse of wisdom. We get truth, says Descartes, when, by searching our own minds, we isolate the "clear and distinct" ideas which reside therein. These ideas and principles are like the axioms of mathematics, in that they are self-evident and necessary. Accordingly, the deductive method employed by the mathematician is stressed by Descartes as the ideal way of securing certainty in our conclusions. This is the way to settle disputes and put thinking on a firm foundation. The empiricist, on the other hand, believing that knowledge comes through the senses, advocates the experimental and inductive method, which is the process whereby the observations which we make are ordered and related into systems. Descartes, like Francis Bacon and Newton, also believed in the necessity for going to nature for the answer to questions about facts, but Descartes never succeeded in reconciling these two interests, the deductive and the inductive.

For a considerable time the views of Descartes dominated both philosophy and psychology, which at that time, and in fact down to the end of the nineteenth century, had not yet split apart. But as time went on Descartes' views became less and less satisfactory. Later thinkers became dissatisfied with the sharp cleavage made between soul and body. Many also found Descartes' theory of knowledge unsatisfactory. However, it was still supposed that in order to have a thorough knowledge of psychology one had to be thoroughly grounded in philosophical training. But times have

changed. In recent years philosophy and psychology have drifted apart in America. Such a statement as was recently made by John B. Watson, that in fifty years philosophy would be dead, would have been impossible for a European psychologist to utter. Inasmuch as European psychology has remained much closer to philosophy than American psychology has, one would expect that European psychology would maintain a livelier interest in the philosophy of reasoning than American psychology. This is indeed the case. One of the best examples of this intimacy between logic and psychology is to be found in the Continental psychological movement which E. B. Titchener [1] has called the *Psychology of Act.*[2]

This notion of the *act* has been the source of much discussion. Because of its significance for logic it will be well to follow the subsequent history of this concept.

Meinong and his followers, known as the Graz [3] school, maintain that stimuli give rise to sensations. This has been a common doctrine in psychology up to the last several decades. Such a question as, "Are there any sensations?" would have been absurd a score of years ago. "Of course there are!" everyone said. According to structural psychology (as this doctrine is called) mental life is composed of a few simple elements of

[1] *American Journal of Psychology,* Vol. 32 (1921), pp. 519-542.

[2] Among the outstanding representatives of this movement are Brentano, Meinong, and Husserl.

Franz Brentano's view, set forth in his book, *Psychologie vom empirischen Standpunkte* (1872), has had a potent influence on later thinkers. His view that reference to an object is the peculiar characteristic of mental phenomena was developed by Brentano's successor, A. von Meinong, in his *Gegenstandstheorie,* or *Theory of Objects.* According to Meinong's doctrine there are three elements in the thought-process, namely, the *act,* the *content,* and the *object.* Professor Edmund Husserl in his *Logische Untersuchungen* also distinguishes between the *act,* the *meaning-content,* and the *object* of thought, which latter may be present or remembered, existent or subsistent.

[3] For a more detailed discussion of these matters the reader may consult Gardner Murphy's *An Historical Introduction to Modern Psychology* or W. P. Pillsbury's *The History of Psychology.*

consciousness possessing certain characteristic attributes. These elements of consciousness, among them *sensations, images* and *feelings,* were said to combine in various ways to produce the more complex mental states. By some sort of mental chemistry these "psychical atoms" were compounded into the more complex "molecular" states. The usual statement of this given by those who accepted this psychological atomism ran somewhat as follows: The child as it first comes into the world is confronted by a bewildering chaos of sensations. Having had no previous experiences these sensations arouse no memories in the child. These sensations may therefore be regarded as "pure" sensations. But as time goes on experiences accumulate, and sensations begin to be ordered. Our sensations are grouped by *acts of attention* into unified patterns of sensations, called *percepts,* which now possess *meanings* for us. A percept, therefore, is made up of what is immediately given, plus supplementary memory images (visual, auditory, kinæsthetic, etc.). According to this account of the development of mental life, thinking consists in combining concepts or ideas according to the *laws of association* (association by similarity, contrast, and contiguity). Thus we develop levels of organization of mental life, where the highest level (of concepts) results from a process of abstraction whereby we draw out from percepts the common elements.

This all seems like clear sailing. We seem to have here a neat explanation of our mental life. Sensationalistic and associationistic psychology had quite a vogue; but its day is over. What brought about the demise of this beautiful theory? The problem which has always faced this type of explanation, which analyzes mental life into its simple elements, is how to get these elements united into the psychic continuum so that we get back that

meaningful world of experience which is after all what the psychologist is trying to explain. How can we get out of such a system that actual order and movement which we are so familiar with, and which William James called the stream of consciousness?

The followers of Meinong, of whom Benussi is one of the best known, maintain that sensations are united into more complex patterns by special productive acts. The Graz configurationists (as they have been called) postulate the existence of a higher psychical activity (*Produktionsvorgang*) which supervises the ordering of the sensory content. Thus the "synthetic unity of apperception," which Kant said was produced by the "transcendental ego," reappears—even though the ego has departed. But has the ego really been banished from the courts of psychology by this scientific decree of expatriation? Bertrand Russell[4] affirms that Meinong's *act* is the ghost of the subject or what was once the full blooded soul. If this is true, we are in this view still not very far from Plato and Descartes.

Finally to lay the ghost of the departed soul is the intention of the newer schools of psychology, particularly behaviorism and gestalt psychology. These two movements arose at about the same time, the first in America and the second in Europe, so that neither has any considerable historical priority. At the present point we will hardly do more than make passing mention of the German movement.

The gestalt psychologists deny that any special process or act, over and above the parts in relation, is necessary to explain the fact that perception and thought are orderly or patterned. This, however, is but a small part of gestalt doctrine. The great value of gestalt theory is that it promises an explanation of the mean-

[4] *The Analysis of Mind*, p. 18.

ingfulness of mental life without resorting to dualistic psychology, and without ignoring as fictitious those realities and processes which constitute our conscious life. The key-concept to the whole doctrine is the notion of *gestalten* (forms or patterns), a concept which those who follow these matters will hear much of in the next few years.

This historical antecedent of behaviorism is the type of psychology advocated by William James. This is known as *functionalism,* a term which contrasts this point of view with the *structuralistic* doctrine which we examined a few paragraphs back. The functionalists, of course, are not the only critics of structuralistic psychology. There has been so much criticism of the analysis of mental states into their psychical atoms that the view has broken down under the combined assaults of various opponents. The best that could ever be said for structuralism was that it was strong on the *descriptive* side (i.e., in the analysis of subjective experience through the highly perfected technique of the introspective method), but this was more than offset by its weakness on the *explanatory* side.

Functional psychologists and pragmatists—including here William James and John Dewey—are interested in that type of investigation which exhibits the rôle of thought-processes in the organic adjustment of the individual to environmental demands. For the members of this school, ideas are not mere froth on the stream of nervous processes, but dynamic determinants in the course of the individual's history. Ideas have motor consequences.

The gravest criticism which can be brought against functionalism is that it does not clarify sufficiently the nature of the relation of consciousness to that part of the body which is the seat of its manifestation. Function-

alism is obviously a reaction against the various types of dualistic psychology which had been put forth. But in insisting that such acts as thinking are causally effective in the behavior cycle we are at a loss to know whether this view, carried to its ultimate conclusion, lapses back into good old-fashioned Cartesian interactionism, or whether it reduces to behaviorism. In John Dewey's viewpoint functional psychology approaches much closer to behaviorism than to interactionism.

And now let us consider more in detail this doctrine called behaviorism. Popularly defined, behaviorism is the psychological theory which states that human beings not only do not possess souls—they are not even conscious. Technically defined, behaviorism is a monism of sensory-neuro-muscular-glandular events, wherein each activity of an organism is explained as a movement of some sort. The behaviorist is not interested in what an individual experiences; he wants to know only what the organism is *doing*. But how, then, does behaviorism differ from physiology? This is a riddle which the behaviorists themselves are trying to solve. The best answer which has thus far been put forth is presented by Dr. A. P. Weiss when he tells us that behaviorism differs from physiology in that it seeks to trace the origin and development of those responses of the organism by means of which it establishes its individual-social (group) status.

The aim of the behaviorist is to make psychology compatible with the universal concepts and methods of science. The behaviorist takes the physical sciences with their objective, quantitative methods and laws of prediction as the ideal pattern of all science. Behaviorism regards organic behavior as a function of the changes in the environment. It will have none of your instincts, "drives," inherited mental traits, or psychic censors.

This environment, as physics teaches us, is a complex of relatively stable physical units of mass and energy. The internal changes in the organism are merely redistributions of the physical energic streams, released by the environmental impingements, along the most permeable pathways.

Viewing both the organism and the environment as configurations of physical elements (*electrons* and *protons,* in the last analysis), the behaviorist finds no need for the hypothesis of consciousness or mental life. He need not deny their existence, but since their existence is not demonstrable by objective observation they cannot be the subject matter for fruitful scientific investigation. Watson contends that it is possible to write a psychology without using the terms consciousness, mind, imagery, introspection, sensation, and the like. Scientific psychology can be written in terms of stimulus, response, habit formation, and habit integration. In brief, human behavior is continuous with animal behavior in every respect except in degree of complexity: we humans are a little more complicated in our machinery, and it is this which makes us a "little lower than the angels."

A few additional remarks about the historical antecedents of behaviorism are necessary. Logically, behaviorism appears to be the inevitable consequence of a strictly monistic interpretation of evolution. It has resulted from an extension of the categories of (Watsonian) animal psychology to the interpretation of human psychology. The advent of the James-Lange (or peripheral) theory of emotion, which defines emotion in terms of bodily manifestations, must be considered as an important incident in the development of a biological interpretation of human conduct. And the increasing insight into the importance of the rôle of glandular secretions

(*hormones*) in personality has seemed to some to fortify the behaviorist's position.

In order to fit thinking into their scheme the behaviorists differentiate between (*a*) *immediate* response and *delayed* response, and (*b*) *explicit* or overt behavior and *implicit* or non-observable behavior. Thinking involves delayed, implicit behavior, largely the incipient movements of the vocal musculature. According to Watson, the alleged reality of consciousness is due to the fact that the subject can phrase in terms of words the habit twists which have become a part of his biological equipment. The "subconscious" and the "unconscious" of the psychoanalysts are names for those processes which are not verbalized. In Watson's[5] words: "The hypothesis that all so-called 'higher thought' processes go on in terms of faint reinstatements of the original act (including speech) and that these are integrated into systems which respond in serial order (associative mechanisms) is, I believe, a tenable one."

In the estimation of Dr. Watson, the most serious obstacle in the way of a free passage to behaviorism is the "centrally aroused sensation" or "image." This question of the existence and potency of the mental image has been such a central problem in modern psychology that we may well digress for a moment and discuss it more thoroughly before continuing.

The question as to whether or not, when we think, images must be present in consciousness has been much debated. If I say, "The pyramids of Egypt have triangular sides," you probably have an image of a pyramid. But is this essential in every case? This is the question of sensory versus non-sensory recall. The problem is

[5] "Psychology as the Behaviorist Views It," *Psychological Review,* Vol. 20, p. 158.

not a new one. Berkeley [6] argued that Locke was wrong when he held that he could form an idea of a triangle without deciding what sort of a triangle it was to be. In more recent times the problem has been subjected to experimental attack. The result of these studies was the development of an "imageless thought" school. In the German movement O. Külpe, head of the experimental laboratory at Würzburg, undertook an examination of the factors involved in "attitudes" (*Aufgabe*)—what we now call "mental set." This work was continued by the adherents of this school—Marbe, Messer, Watt, and others—who also came to the conclusion that the sensory element in thinking had been exaggerated. In France, Binet [7] came to the conclusion that images form only a small part of the total phenomenon of thought. Some times, Binet states, the series of images is not even consistent with the thought. So we see that on the whole Berkeley's view has been substantiated in this respect.

Thus, as Watson believes, one of the greatest obstructions to the behavioristic program has been removed by the dismissal of the image from psychology. Watson's attitude towards the image has been best stated in his reply to Russell's criticisms of behaviorism in his book, *The Analysis of Mind.* Watson's argument [8] runs as follows: The behaviorist believes that what most psychologists call the "image" has as definite a cause as "sensation." He has no trouble in finding a means for providing an actual visual stimulus as part of the complex of stimuli which arouse a total reaction to an object not present to the senses. Knight Dunlap, although not a behaviorist, first pointed the way by claiming that the so-called visual image is only an associated eye muscle

[6] *A Treatise Concerning the Principles of Human Knowledge,* Introduction.

[7] "La pensée sans images," *Rev. Phil.,* Vol. 28, pp. 138-152.

[8] *The Dial,* Vol. 72, pp. 97-103.

strain. "In other words," as Watson says, "when you are thinking of a definite object (not present to the senses) which the eye has been trained on, 'imaging' it, the eye muscle adjustment takes place actually (though faintly) as though you were seeing the object. The behaviorist without giving up his premises—to the effect that a sense organ stimulus is always present in any reaction—admits the associated muscular adjustment of Dunlap and also conditioned reflex eye muscle responses (which may have a different origin from the associated), and if necessary can go still further and say that the associated and conditioned reflex muscular responses in the eye may bring about just enough tension upon the eyeball, and hence upon the retina, to start faint retinal activity. It is well known that phosphenes, rings of light, 'stars,' flashes of light, can be produced by pressure and by electric stimulation of the eye. The behaviorist can go still further and maintain that in a normal person the retina is a sense organ of such delicate chemical and physical structure and balance that optical sensory impulses are always passing towards the brain. He might even argue that centrifugal nerve fibres keep the retina constantly stirred up and supplement in this respect the work done by associated eye muscle responses in causing the arousal of actual retinal impulses." It will be observed that in introducing these ideas Watson is almost abandoning the notion that thinking is sub-vocal speech.

Everyone who has become entangled in the briers of modern introspective psychology, who has become stuck in the quicksands of subjectivism, can sympathize with the behaviorists when they propose to avoid these obstructions. It is true that when we get to the so-called higher mental processes (or the simple, for that matter), the physiological bases become so obscure, the alleged experimental results so conflicting, and, in general, the prob-

lems so complex, that one is tempted to abandon the ship. Such an apparently simple phenomenon as reasoning involves questions concerning the realization of purposes, foresight of ends, the nebulous notion of intelligence, the problem of volition, and other intricate phenomena of such an intangible nature as to frighten off all but the most stubborn. Ethical considerations—problems concerning hedonism, etc.—also enter in to strengthen this fear complex directed against metaphysics. In behaviorism all these problems are swept aside. *The behaviorist,* in short, *is a person who believes in the magical efficacy of a fresh start.*

It is not the business of writers of books on logic to tell the psychologists what kind of psychology they ought to advocate. But on the other hand, the logician does have the right to insist that whatever psychological theory may be developed, it must provide an adequate basis for what the logician has to deal with—such phenomena as judgment, inference, meaning, and the like. It seems clear that it is precisely at this point that the defects of behaviorism become most manifest. Any theory which lays claim to being "true" must be prepared to state what truth is in terms of its own system. We have yet to see a behavioristic definition of truth. And why should the behaviorist regard his reactions as any "truer" than the verbalized responses of a Christian Scientist? Are they not biologically on a par?

The behaviorists frequently accuse other persons who do not accept their doctrine of not *understanding* the meaning and the implications of behaviorism. But what does "understanding" mean in the behavioristic system? Can one understand a theory without believing it? Common sense states that you can, and presumably the behaviorist is still sufficiently "normal" to agree with common sense. But what, in terms of neuro-muscular events,

is the difference between *understanding* a proposition and *believing* it? Belief is ordinarily interpreted to refer to the attitude taken by the subject towards the proposition which is understood. According to Brentano, one can apprehend the meaning of a judgment without affirming its truth or falsity. But how can this "independent variability" of meaning and belief, of cognition and conation, be interpreted in behavioristic terms? We are still awaiting the answer to this and to other questions.

It may be that the absence of satisfactory answers to these questions does not stand as a permanent criticism of behaviorism. Perhaps answers will be forthcoming in the future. For that matter, other types of psychological theory are hardly any better off so far as their ability to answer these questions is concerned. But these other views do not attempt to steer clear of these problems simply because they are so difficult and have therefore thus far defied solution. Perhaps if the behaviorists would turn to these problems and give them serious consideration they would in time be able to give us satisfactory answers to some of these questions.

Chapter VII

THE EVOLUTION AND FUTURE OF LOGIC

FOR some curious reason the history of logic has been neglected. Only the Germans have manifested any keen interest in this field. If one examines the current books on the subject no satisfactory discussion of the evolution of this discipline is likely to be found. Those treatises which do contain a history of the subject seem to regard the introduction of this historical material as a necessary evil, to be hurried over as quickly as possible.[1] If one believes the texts, logic is a science which was developed mainly by three men: Aristotle, Francis Bacon, and John Stuart Mill. Furthermore, the impression one carries away from reading some of the discussions of this subject is that logic is completely lacking in the kind of romance which might be expected to inhere in a subject concerned with man's attempt to understand the mysteries of the world in which he lives.

But surely this is to overlook the possibilities of the subject. Such short-sightedness must have come about as a result of the tendency to abstract logic from its concrete settings and cut it off from the roots which run deep into the soil of social culture and human nature. No matter how abstract logic may be in its ideal form (as the science of *order* and *systems*), if beginning students are to profit by their study introductory discussions ought to relate logic to life and experience. Let us see whether this situation can't be remedied.

[1] Exceptions to this statement are to be found in J. E. Creighton's text, *An Introduction to Logic,* and A. E. Avey's book, *The Function and Forms of Thought.*

The further back one goes into the history of human thought the less specialized are the arts and crafts and the intellectual pursuits, such as they are. This fact is so well known that one hardly needs to point out, for example, that in primitive thought religion, art, philosophy, and science do not exist as separate and independent interests. Accordingly, as we would expect, if we go back far enough we come to a place in human thought where it is impossible to talk about logic as logic, and science as science. As we have already seen, there is a question as to whether, even now, we can separate logic and mathematics. How much more difficult, therefore, it is to try to separate them in earlier times when these subjects were much less specialized. Consequently, in discussing the early history of logic, we will not attempt to distinguish between logic and mathematics.

We are here dealing with thought at its most primitive level, undifferentiated and untutored. What picture can we form of primitive man, who has not inherited that system of principles called the science of logic, as he faces the dangers of a sometimes hostile but always mysterious environing world?

The process by means of which man learned the principles of correct inference (as we now understand them) may be compared to the process by means of which a child learns to walk. Indeed, there is something more than mere analogy here. When the ancestors of the human race ceased to go on all fours and stood erect the earlier mode of locomotion was abandoned. Standing erect freed the hands from the need to use them for hanging on to the limbs of trees. This made possible gesture, which became the ancestor of sign language. But later, with the fabrication of stone and iron tools, man found an additional use for his hands. It then became desirable to use some other instrument than the

hands for communication, so the vocal organs were specialized to take over this function. But primitive man still found his hands, particularly his fingers, useful for counting. Now, in modern times, our thinking, like our bodies, is supposed to stand erect on its own feet.

Dr. A. N. Whitehead [2] tells us that it must have taken the human race ages before men learned to think in terms of abstract numbers rather than in terms of concrete objects. It is not known how old the human race is. But if we go back only so far as the Cro-Magnon race (twenty or thirty thousand years ago) we will probably find that this race committed very elementary logical blunders. Science is but a very recent addition to culture —some sciences are still in process of being born. This is not to say that ancient man had no *culture*. But he did not possess *science*. To be sure, some persons who like to glorify the past talk about the "lost" arts, but it is doubtful whether the ancients possessed any secrets of importance which we do not now possess.

It is difficult for us to reconstruct the mental life of a savage. The difference between primitive man and modern man is a result largely of the tremendous advantage which our inherited social culture has given us. It is not easy for us to understand that primitive man had no definite concepts of space, time, causal relations, and the like. Such a relatively easy operation as addition or multiplication was beyond the most enlightened early primitive man—he had to learn the science of numbers by creating it. And he probably experienced much more difficulty in doing this than does the child in the grade schools today.

Where did the first beginnings of science occur? If we agree with C. J. Keyser that mathematics is the most ancient of the sciences, and if we regard logic in its

[2] *Science and the Modern World*, p. 30.

earlier form as a branch of mathematics, this problem resolves itself into the question of where mathematics first appeared. If we consider number and geometry as branches of mathematics, as of course they are, we find the experts answering our query with the statement that number is as ancient as spoken language and probably older than any system of written language. But geometry, which must be considered as a form of reasoning, is of more recent origin, having been first developed to any considerable degree by the Egyptians.

The first civilizations to appear (in the Occident) were along the fertile river valleys. Centuries before the Chaldean and Assyrian civilizations along the Tigris and the Euphrates, there was developing along the Nile river the culture of the Egyptians, to which the pyramids stand as a fitting monument for all time. Undoubtedly the Egyptians possessed some knowledge of physics and mechanics, and of course they were much interested in astrology, alchemy, and magic. According to G. Elliot Smith, who is the outstanding advocate of the doctrine of the *diffusion of culture,* the civilization of the whole world was inspired by Egypt. Not only Crete and Syria, but Mesopotamia and India, Africa, and Europe, and indirectly the rest of the world, even including America, derived their cultural capital from this cradle of civilization—according to this British anthropologist. Needless to say, many other students of these problems are not willing to accept this conclusion.

Mathematics and logic occupied the same position among the old Egyptians, Babylonians, and Assyrians that philosophy did in Europe during the period of the scholastics—it was the hand-maid of religion. Among these peoples astronomy, magic, and geometry were subordinate to utilitarian interests.

Herodotus and Plato ascribe the origin of geometry

among the Egyptians to the annual inundation of the Nile. In overflowing each year it wiped away the old landmarks, thus making it necessary to resurvey the country in order to avoid land disputes and determine taxes. As early as 2000 B.C., according to one authority, the Egyptians were familiar with the properties of the right triangle, whose sides are to each other in the ratio of 3:4:5. This fact was employed by the "rope stretchers" in their surveying of the land.

The Egyptians contributed little to the theory of any science. Abstract science is the great gift of Greek genius to civilization. The extent to which the Greeks were indebted to other cultures is disputed. There are those who argue that Plato was familiar with Babylonian ideas. It is supposed by some that Pythagoras was familiar with the ideas of the Hindu mystics. As we have seen, G. Elliot Smith believes the Greeks are indebted to the Egyptians for many of their ideas. But in these matters the truth is very hard to discover. If the Greeks did borrow, they also improved and added original contributions. The great superiority of the Greeks is illustrated by the way in which they establish as a general theorem the proof which, in honor of its supposed discoverer, is called the Pythagorean theorem. In other words, while the Egyptians thought in terms of the concrete and specific, the Greeks possessed the unique gift of being able to establish universal principles which were good for all cases of a specific class.

Here we discover the general rule covering the development of logic: *from its most primitive beginnings to its latest and most abstruse flights the development of logic is in general a movement away from concreteness towards greater and greater abstraction, with the parallel increase in the facility and efficiency in use of a symbolism which represents these logical concepts and processes.*

The very early Greek philosophers, Thales, Parmenides, and others of about the fifth century B.C., probably sensed the necessity for what we now call the laws of thought. Such formulæ as, "out of nothing nothing comes," and "non-being is not" are probably the ancestors of what we now call the laws of thought.

The first important figure in the history of logic is Socrates (469-399 B.C.). The main contribution of Socrates was the doctrine of the *concept*. The *Socratic* method was merely the technique Socrates employed to transcend the particulars of sense experience and get at *conceptual knowledge*. Since we will have more to say about perception and conception, and since we will also discuss in more detail the view which is known as scepticism (which Socrates and Plato were trying to refute), we will not say anything further at the present time about these two twin stars of classical Greek thought.

In Aristotle (387-322 B.C.) Greek thought in the field of logic reaches its zenith. The various contributions of this versatile and powerful mind were gathered together by later students into the *Organon,* that is, the *Instrument* of thinking. As is well known, Aristotle gave us the *syllogism,* the typical form of deductive argument. Little has had to be added by later scholars to Aristotle's contributions in this field. The famous tutor of Alexander the Great did for deductive logic what Euclid did for geometry: he systematized and generalized and added greatly to the contributions of earlier men, putting upon the science the stamp of his own vigorous intellect.

After the breakdown of Greek culture no contributions of importance were made to logic until comparatively recent times. The Romans and the Medieval scholars of Western Europe were content to live on the heritage which had been bequeathed them by the Greek intellectual titans. The Stoics made some contributions to logical

theory which were of minor importance. Cicero was interested in the application of logic to rhetoric; but no really fundamental work was done.

The system of Medieval thought which is known as Scholasticism, and which has its best representative in Thomas Aquinas in the thirteenth century, employed the deductive method of Aristotle almost exclusively. The reason for this is easily seen. If you assume that all knowledge is given (in the synthesis of Greek philosophy and Christian theology), all you can do is see what else can be inferred from this body of presuppositions by making deductions from them. Obviously you cannot get any new knowledge; but this did not perturb the Medievals, for they believed that everything worth knowing was already known to them. Hence their total lack of interest in nature and what we now call inductive science. There were a few thinkers—such as Roger Bacon—who attempted to introduce observation and empirical method; but they were incapable of breaking the chains of authority.

René Descartes (1596-1650) is regarded as the founder of modern philosophy. His method of *systematic doubt* constituted a notable advance in human thought, inasmuch as it tended to replace faith by reason as the sanction for belief. Descartes' interest in the natural world is illustrated by his various theories in the field of mechanics and cosmology. Descartes respected reason more than tradition and authority; had he but come a little later he would undoubtedly have been a great physicist. As it is, he laid the foundations for analytical geometry, which was employed so effectively by Sir Isaac Newton and others.

The next great figure in the history of logic is Francis Bacon (1561-1626), who is commonly supposed to be the founder of inductive logic and the great proponent

of experimental science. Bacon's supposed originality in this field has probably been exaggerated. Moreover, the problem of inductive inference has turned out to be much more complicated than Bacon ever imagined it could be. Nevertheless, Bacon did much to explode the pomposities of Medieval thought, so that he still deserves his place in the hall of fame. Francis Bacon's ideas were developed into a technique for establishing causal laws by John Stuart Mill, a great humanitarian and political scientist as well as a brilliant logician. We shall have occasion to employ Mill's *canons of induction* later on.

The program for what is now called *symbolic logic* was first outlined by the German philosopher and mathematician Leibniz (1646-1716), though there were anticipations of such a project in Plato and Descartes. Leibniz's contributions are scattered through many different manuscripts; owing to the fact that many of these were (and still are) unedited some of the ideas which Leibniz put forth had to be rediscovered by later investigators.

Leibniz, who along with Newton invented the infinitesimal calculus, possessed the mathematician's method of attack. He believed that all ideas were combinations of a small number of elementary ideas. If each of these ideas is symbolized by a character (or "ideograph") which would be the same the world over, we can construct a universal alphabet of human thought. This universal language (or *characteristica universalis*) for the expression of ideas was to be supplemented by a calculus of reasoning (*calculus ratiocinator*) which would exhibit the universal relations among such concepts or ideas. By making the various possible combinations according to the rules of construction Leibniz believed that we could anticipate all possible knowledge.

This was indeed a fertile suggestion, but unfortunately the idea failed to grow. It remained for George Boole

(1815-1863) and Augustus De Morgan (1806-1878), two Englishmen, to formulate the fundamentals of a logic of relations and a calculus of reasoning. Following these individuals other investigators attempted to develop and improve upon the algebra of logic. Charles S. Peirce in America, Ernst Schroeder and others in Germany, and Whitehead and Russell in England have continued the line of development started by Leibniz. At the present time the future of this system appears very promising.

In the foregoing pages we have observed that logic as a science has developed; the logical principles we now employ are products of the evolution of the human mind. Now we must consider the future of logic. We must consider the question which we have touched upon in previous chapters: Will logic continue to evolve in the future as it has in the past? Or has the human mind reached its full maturity? Let us see!

The Future of Human Reasoning

We are told that armies advance on their stomachs. One might also argue that the same mode of locomotion underlies the progress of civilization. No doubt the fastidious would object to the application of such a statement to the cultural advance of the race. And yet there is a sense, figurative perhaps, in which this somewhat inelegant phrase is quite appropriate. Consider for a moment the progress of the caterpillar. It consists in pushing the head end forward and then drawing up the rear end to the place where the head end formerly was. The progress of civilization is similar to that.

We find that as human culture evolves and our ideas change a twofold process is at work. On the one hand, we observe a small group of individuals engaged in criticizing old ideas, perhaps finding them wanting, and attempting to create new models of thought. At the

same time, there is a larger group of individuals engaged in pulling up the rear end of the social caterpillar, educating the stragglers to the point where they can assimilate what has already been achieved. At the present time the business of education seems to have become a matter largely of bringing up the rear end. Education for the most part consists in passing on the standardized patterns of behavior and thought.

When we consider in detail this matter of pushing the head end forward we discover that in so far as we are dealing with reason there is little evidence of advance since the time of the old Greeks. As we have seen, Leibniz attempted to formulate a kind of calculus of reasoning which was the ancestor of what has developed into mathematical logic. But this type of logic has not, up to the present, interested many persons, and certainly it has not caused any fundamental modifications in our habits of thought. We may therefore conclude that so far as *methods* of reasoning are concerned human thought has stood still for over two thousand years. The only essentially new idea, mathematical logic, has failed to produce any effects on the habits of thought of men.

When one inquires into the reasons for this lack of progress since the time of the old Greeks, several possible causes suggest themselves. For one thing, thinkers have been busy applying the Greeks' heritage of logical method to the solution of those problems which were also taken over from Greek culture. Philosophers have been too much occupied solving these problems with the tools already available to look for new instruments of thought. Another reason may perhaps be found in the fact that the public has not felt or expressed a need for additional or novel principles of thought. The social stimulus was wanting which might otherwise have called forth from logicians an inquiry into the possibilities of other types of

reasoning devices besides those already available. The public in general, along with academic philosophers, had all they could do to organize and assimilate the existing logic and bring it to bear upon the solution of problems already in existence.

This application of traditional logic to the pretended solution of the supposedly real problems of theology, philosophy, and science may have been unfortunate for the human race. Bertrand Russell has on several occasions declared that he doubted whether anyone trained in Aristotelian logic could ever free himself sufficiently from that training to think clearly—to think in terms of mathematical logic. By implication Mr. Russell is therefore saying that the human race (or at any rate the civilization of Western Europe) has been corrupted by its devotion to traditional habits of thought. Here we might glance at Mr. Russell's views concerning the tyranny of language. In his book, *The Analysis of Mind* (p. 212), Mr. Russell argues that many philosophers have erred in assuming that the structure of sentences corresponds to the structure of facts. He here refers to the doctrine of Sayce, who maintained that all European philosophy since Aristotle has been dominated by the fact that all philosophers spoke Indo-European languages, and therefore supposed the world, like the sentences they used, necessarily divisible into subjects and predicates. Along somewhat the same lines, we have the view of Dr. A. N. Whitehead that the dualism of substance and qualities is a result of the dominance of the Aristotelian logic of classes (of subject and predicate terms).

Bertrand Russell's wholesale condemnation of logical traditionalism is an illustration of the type of impatience which heretics have always had for accepted customs and

conventions. But, interestingly enough, those who have tried both methods of teaching logic usually find that most students who make an attempt at understanding traditional logic are successful, while those students on whom mathematical logic is imposed make little headway in understanding or in using it. One possible explanation of this may be found if we accept the organismic analogy. In what is known as the culture-epoch theory we have a psychological doctrine of mental evolution which is similar to the biological doctrine of embryological recapitulation. According to this theory, the human mind, as it evolves in the individual, retraces the intellectual evolution of the race. If there is a valid analogy between the two processes of recapitulation (bodily and psychic), it may be that human beings find it so difficult to think in terms of any other doctrine besides traditional logic because in thinking in accordance with the logic of Aristotle, Bacon, and J. S. Mill we are merely reasoning along the lines towards which we are naturally predisposed by our mental heredity.

Whether this explanation be true or false, it is evident that up to the present time we have, in so far as we have reasoned at all, been content to think according to the patterns preëstablished by the geniuses of bygone ages. But now things are changing. Logicians are waking up from their dogmatic slumbers. Indeed, they are being shaken from their stupefaction by scientists, physicists for the most part, who have been bitten by the virus of criticism and dissatisfaction which swept the world after the war. In other words, the demand for a fundamental consideration of the nature of our reasoning processes and the influence of logical presuppositions on scientific formulations is coming as much from science as from within the field of logic itself. One illustration of this appears in the following situation.

It is generally known that physicists at the present time are face to face with a situation which calls for two apparently opposite modes of thought. The wave theory of light and the corpuscular theory both explain some of the observed phenomena, and yet these two seemingly opposed points of view are very hard to reconcile. Sir William Bragg has suggested that here we have an actual case of where the human mind is confronted by its own defects. Several writers have suggested that what is called for here is a new mode of thought. Professor F. A. Lindemann,[3] developing the possible consequences of the new type of algebra which has been applied to quantum phenomena, states the situation in these words:

> It is early yet, if not too early, to proceed further. Scarcely free as we are from the shackles of Euclidian geometry it is rash perhaps even to envisage the next step. For the conventions and sanctions which bolstered up Euclidian space are as nothing to those which will be invoked to maintain inviolable the sanctity of logic. Yet the prediction may be permitted that even this stronghold of the formalist will one day be invaded. If non-Euclidian geometry has been shown to be possible and even applicable to physics, if arithmetic in which 2×3 is not equal to 3×2 could be developed, and has even proved adapted to physical calculations, can it be maintained that an alternative to our ordinary logic is unthinkable? The attack upon the citadel will indubitably be launched. Whether it will prove successful the future alone can tell. For more than a hundred generations defences have been subconsciously prepared. The most important perhaps is language. Logic, indefinables and mental habits of thought have molded it and have been moulded by it to such a degree that it is almost impossible to escape from the tyranny of its perpetual suggestion.

In suggesting that the new spirit of criticism has to a considerable degree been forced upon logicians by outsiders we do not imply that logicians have been the victims of mental anæmia. We are merely pointing out the interesting fact that science is giving added impetus to a

[3] In his essay on "Physics" in the coöperative volume, *The Mind.*

movement already under way. Recent years have witnessed what one writer describes as a renaissance of logic. Part of this renaissance includes a recognition of the changing and imperfect character of the science of logic—something, by the way, which was quite foreign to the spirit and the thought of the Medieval world. But this view is of recent origin, and there are those who maintain that if growth is to be admitted in the science of reasoning it must come at the top and not at the bottom of the tree of knowledge.

One way of surveying some of the new developments in logical theory is to state the principles which have been accepted as foundational to logic and then indicate the various proposed modifications of them which have been offered. The following, as we have previously noted, seem to cover the fundamental assumptions which have been commonly accepted in the field of logic: (*a*) the *law of identity;* (*b*) the *law of contradiction;* (*c*) the *law of excluded middle.*

These laws are (or were) supposed to hold throughout the entire domain of logic, inductive as well as deductive. It will be noted that they are termed *laws,* which implies that they are considered to be not merely convenient assumptions, but necessary and inescapable axioms of all thought. But if the criticisms which some are now urging against these so-called laws of thought prove to be valid, we may be forced to conclude that these laws may not be necessary or logically inevitable. To decide this matter we must examine the criticisms of the laws.

Many modern thinkers have been captivated by the notion of evolution. With Heraclitus they affirm that all things flow, that nothing is permanent. This evolutionary view is frequently interpreted to imply the rela-

tivity of all standards. If everything is in flux then certainly the standards of morality and perhaps even the so-called laws of logic, by which the mind has guided its reasoning processes in the past, are all evolving, are not absolute, and therefore will be superseded by other principles. Thus argue the modern disciples of Protagoras, the ancient Sophist.

But up to the present time the evolutionists and relativists have not seen clearly just how they might go about demonstrating the relative character of the laws of logic. The absolutist (as he is sometimes called) has always pointed out that when you say everything changes, that nothing is permanent, you implicitly make an exception of that proposition; it, at any rate, falls outside the processes of change. You affirm that this law of change is eternally true, always the same, yesterday, today, and tomorrow. In other words, the absolutist or eternalist argues that the evolutionist and relativist contradicts himself and thus refutes his own doctrine. Josiah Royce, who had numerous arguments with William James on these matters, put the position in this form: There are certain truths so fundamental that the very attempt to deny them reasserts them. And, he points out, when you come across a proposition which is so fundamental that the very attempt to deny its truth really reinstates it, then you have absolute truth. If I am not mistaken, this idea was first presented by Leibniz.

Obviously, if the modern advocates of logical relativity are to make good their claims they must be able to produce other postulates according to which reason can proceed and arrive at satisfactory results which enable the organism to get on in the world. For example, the relativists must show that they do not have to reason according to the law of identity, the law of contradiction, and the law of excluded middle. And to refute the new

logic it would be necessary for the absolutist to show that, in spite of himself, the logical relativist uses these very laws which he proposes to expunge.

It is interesting to note that a follower of John Dewey, Dr. B. B. Bogoslovsky, has proposed that we substitute a new law for the old law of excluded middle. In his book, *The Technique of Controversy,* he criticizes the old law as being devised for a static world. In a world which does not evolve, where everything is always the same, it is evidently true for all time that plants are plants and animals are animals, and "never the twain shall meet." But, argues the innovator, in an evolving world plants become animals. The A that was B becomes not-B, and obviously at the point of transition A is both B and not-B. This fact that when a thing changes it ceases to be itself (i.e., what it was) and becomes something else Mr. Bogoslovsky proposes to take care of in his law of *included middle.* This is not the first proposal which has been made to abandon this law, but since the implications are worked out in the greatest detail by this investigator we will confine our remarks to his statement.

What reply does the absolutist make to this proposed revision? There are several avenues of attack upon this logical heresy. In the first place, it may be pointed out that merely to reason about an evolving world does not require a new logic. The old Greeks and the modern followers of Darwin have reasoned about an evolving world of organisms without feeling the need of a new logic. Moreover, the recognition that the A that was B can become not-B is not the exclusive vision of the admirers of John Dewey. In the New Testament it is recorded that Jesus points out that while men say this is this and that is that, He declares that *this is that!*

Still more specifically, the absolutist will point out that this doctrine really presupposes the logic it is rebelling against, and is therefore self-refuting. If the new law of included middle be accepted as true this means that an appropriately chosen proposition—such as that Jesus was a pacifist—can be both true and not true. Another way of putting this is to say that two contradictory propositions can both be true at the same time. If we abandon the laws of logic which prohibit this, the absolutist declares, intellectual chaos results. Consistency is impossible in a world where contradictory propositions are all true. But obviously those who argue for this new logic do not mean to be inconsistent. Hence, say the defenders of the old logic, those who try to persuade people to abandon the laws are appealing to them. When, for example, Dr. Bogoslovsky rejects the law of included middle he is virtually saying that he is putting this law in the class of propositions which are not true. But this clearly presupposes that every meaningful proposition is *either* true or false, *but not both.* On the other hand this logic is supposed to make it possible for propositions to be *both* true and not-true (or partially false) at the same time. This is certainly a contradiction. The person who says that the law of excluded middle is not true is assuming the very thing he is denying.

This view of the absolutist does not exclude the possibility that a proposition which is true or false at one time may become false or true at some later time. Nor does it deny that a proposition which is true in one sense may be false in some other sense. The traditional laws of logic do not prohibit either of these possibilities. And the assigning of "truth-values" to propositions, thus introducing probability considerations, does not conflict with the old laws. The laws state that the same proposition cannot be both true and false *in the same sense,* at

the same time. The law of identity tells us that *A is A*. This means that the meaning of a word or a proposition must remain identical with itself throughout its use in a given "universe of discourse." Violations of the rule occur commonly enough, but whenever we note them we point them out and seek to abolish the duplicity of meaning. Sheer madness would result if a word meant what it does not mean. To be sure, in a world which progresses mentally words do change their meanings. But the admission of this fact does not upset the law of identity.[4]

Conclusions

The foregoing survey seems to have demonstrated that if our reform in logic is to start with the three laws of logic the prospects for evolving a new logic do not appear so very bright at present. Admitting that there is still considerable room for further investigations in the foundations of logic, we must also concede that traditional logic, further generalized through the contributions of mathematical logic, seems to provide a satisfactory basis for human thought and action. To appeal to the case of geometry, where the view that the axioms of Euclid were logically inescapable was later refuted by the work of the non-Euclidian geometers, overlooks the fact that logic is more fundamental than mathematics.

[4] It is evident in our discussion that while the three laws of logic are here regarded as being to some extent independent of each other, they are also considered to be related to each other. This suggests the possibility they all three stand or fall together: if you accept one you accept all, and if you reject one you must reject all. Some experts will not agree to this. Mathematicians like L. E. J. Brouwer have proposed to abandon the law of excluded middle, or the law of contradiction (the law of excluded middle or the principle of *tertium non datur* in Brouwer's case), while yet retaining the others. But it must not be overlooked that Brouwer is working in the field of *transfinite* numbers (the "theory of aggregates"), where the ordinary rules of arithmetic no longer hold. It is therefore doubtful whether Brouwer's views are relevant to our present problem.

In logic we study the ways in which reason must proceed in any field whatsoever—mathematics (Euclidian or non-Euclidian), theology, natural science, and so on,—if reason is to reach valid conclusions. Logic, in other words, is unique; it deals with a purely ideal realm of eternal and changeless logical atoms—terms and relations—which incidentally are (or can be) applied to what Bertrand Russell calls this higgledy-piggledy job lot of a material universe in which we live. This discipline studies what may be called the logical invariants of all possible kinds of thought. Its existence seems to be grounded in a system of *super-postulates* (to use Dr. H. M. Sheffer's term), assumed in all possible kinds of reasoning.

Part III

HOW OUR KNOWLEDGE IS ORGANIZED

Chapter I

THE CYCLE OF THOUGHT

THERE is a vast difference between the simplest forms of organic creatures and the most highly developed. The difference between an amœba, a unicellular organism, and man, the most highly specialized of the multicellular organisms, is a difference in number and degree of complexity of parts. These differences in structure make possible differences in function or type of behavior. Simple organisms are capable of but few and relatively simple types of response. The behavior of the higher organisms, those which come latest in the evolutionary development of living forms, is very complicated. Fortunately, it is possible to classify these different modes of response into two main groups. Any given response of any organism may then be put into one or another of these several classes of behavior. These two types of response are as follows:

Innate Behavior
- *Tropisms*—e.g., heliotropism
- *Reflexes*—e.g., the pupillary reflex
- *Instincts*—e.g., fighting, sex, etc.

Acquired Behavior
- *Habit*
- *Voluntary Behavior*
- *Reasoning*

Let us first consider the type of conduct included under the term, innate behavior. Responses which fall within this class are automatic or mechanical, in the sense that they are "ready-made"; they are not the result of conscious deliberation. Furthermore, these inherited or un-

learned forms of behavior cannot be modified to any great degree. Such responses require no training for their performance. The behavior of lower organisms is almost entirely of this sort. Such creatures profit little by experience, are relatively unintelligent, and cannot be educated in any great measure.

Organisms characterized by behavior which falls largely within the second class of responses are much more variable in their responses. They can acquire new forms of behavior in addition to those which are inborn. Human beings, who function largely on this second level, are more the products of the social environment than they are of biological heredity. Reasoning, purposive behavior which is consciously initiated and guided, and intelligence, are the distinguishing features of this type of conduct. This type of behavior is non-automatic; even habits, which are normally performed without conscious control, are subject to modification: we can break old habits and consciously learn new ones, if we so decide.

While reasoning and intelligent behavior fall within this field of consciously initiated and controlled response, it does not follow that all processes which are conscious are reasoning processes. Sensation and perception, for example, involve consciousness, but they are not considered to involve reasoning processes. Indeed, none of the so-called elementary mental processes involves conscious thought. Reasoning is restricted to the higher mental processes, where ideation is present. Ideas are the *sine qua non* of thinking or reasoning.

One of the most enlightening differentiations between different types of ideational processes is embodied in the distinction between the following two classes of mental association:

(A) *Free Association of Ideas*
(B) *Controlled Association of Ideas*

The first type of thinking, or ideation, is what we mean by daydreaming, "wool gathering," or "building castles in Spain." Here one idea calls up another, but there is no attempt made to guide the train of thought along some preëstablished track. Such thinking is pleasing for its own sake: it satisfies an emotional need, but not a logical demand. This type of thinking has appropriately been termed *compensation* thinking. Some psychologists call this *autistic* thinking. In such thinking the wish is father to the thought, so that in this ideal world, molded according to the heart's desires, we make up for the shortcomings of a niggardly environment of reality.

In controlled association of ideas there is rigorous criticism of the ideas which present themselves to consciousness: not any idea can then get into the mind (or pass by the "psychic censor"), but only those which bear their logical passports with them. In logical thinking there is an intellectual vigilance exercised, so that only logically relevant ideas are permitted to pass. The control exercised is related to the fact that we have a definite problem confronting us to which we are trying to find the answer. In such cases there is a greater sense of effort present, and guiding the train of thought is soon likely to become hard work. This kind of thinking, in contrast with compensation thinking, is called *reality* thinking by some psychologists.

Freudian psychology, as we have already noted, has tried to show that most (if not all) our thinking is "rationalization." Originally this term was not one of disparagement; but now to be accused of rationalization is regarded as a condemnation. At first rationalizing meant finding reasons for, or justifying intellectually, a course of action or a belief. Now it has taken on the meaning of finding an excuse for what we are already predetermined to do because of the dominance of emotion

and instinct. The "new" psychology tends to depreciate reason, making it the slave of the desires, or the *libido*.

The *reductio ad absurdum* of all such theories is found when we apply these doctrines of reasoning to themselves as illustrations of the reasoning process. In the case of the doctrine that all reasoning is rationalizing, we may ask the question: What particular emotion or complex is the Freudian theory satisfying when Freud enunciates his theory? The actual fact is that Freud and other psychoanalysts present their theory, not as a rationalization of their own cravings, but as reasonably true doctrines. But this is equivalent to admitting that the Freudian theory does not apply to Freud's theory, that is, to the reasoning processes by means of which Freud arrived at the Freudian theory. Thus reason recovers its original power as an instrument for discovering truth.

Another criticism which may be brought against theories which minimize the importance of reason in the individual and in society is that they fail to take into account the *creative* character of reason. This point deserves emphasis. This fact is brought out in the following scheme,[1] designed to illustrate the difference between the several types of processes which are commonly classed together under the term, cognitive processes:

Memory.....*True,* but not *New*
Imagination.....*New,* but not *True*
Reason.....*New,* and accepted as *True*

It is an interesting problem to discuss whether, and, if so, in what sense and to what degree, animals think. We so frequently read or hear of the unusual intelligence which animals appear to possess that one might suppose that the common belief is that animals do possess unusual intelligence. But, curiously enough, popular belief seems to be divided on the matter. Even in the

[1] Taken from W. B. Pillsbury's *Essentials of Psychology,* p. 217.

same individual there is sometimes the tendency to exaggerate the powers of reasoning in animals (particularly pets or domesticated animals), while yet denying that animals are sufficiently similar to human beings to justify the conclusion that human intelligence was evolved from the "animal mind." At the present time the tendency in psychology is certainly in the direction of admitting that the animals biologically closest to man are "almost human." The old belief was that animal learning was by "trial and error" (mostly error), but Köhler's [2] work on apes seems to show that they are capable of using *insight* in the solution of their problems. But there still remains a gap between the type of problems which animals can solve and the type of problems which human beings can solve. This difference is apparently due to the fact that human beings can form abstract ideas (concepts), whereas animals are limited to perceptual associations. This difference is due to the fact that human beings possess in a superlative degree (as compared with the lower animals) the following two qualities: (*a*) a superior capacity of recall or memory, and (*b*) the ability to communicate through the use of articulate speech. A few words about each of these will bring out their importance.

It is no accident that those animals which have the longest memory spans are the most intelligent. Animals *take in* and *give out,* while man takes in, *works over,* and gives out. This working over is what makes it possible for an individual who is facing a new situation to delay response, reflect, call upon his past experiences (this latter involving *individual* memories rather than the *motor* or habit memory of lower animals), and solve the new problem by bringing to bear this funded wisdom which he has accumulated in the past.

[2] *The Mentality of Apes,* by Wolfgang Köhler.

We have already discussed at some length the matter of the relation of articulate speech (language) and thought. Man lives in a world of objects, like any other animal; but in addition he lives in a world of words. It is because he can use words as symbols that man has the power of thinking in terms of abstractions, such as "justice," "beauty," "truth," and the like. Part of the biological explanation of man's mental superiority is to be found in those mutations of the vocal organs which enable man to articulate words, subvocally as well as aloud. But this is not the whole story. Some animals—the parrot, for instance—can be taught to talk, but no one stands in awe of their powers of reason. Undoubtedly the addition of several hundred of grams of brain material which man possesses must be highly important in creating the gap between the species *homo sapiens* and the anthropoid apes. But the understanding of how the cerebrum makes possible the functioning of the human mind is still to be sought, and the logician must wait upon the biologist and the psychologist to provide the solution.

The logician's problem is to take the mind as he observes it to function in an intelligent organism and trace out the operational processes by means of which this creature tackles novel situations, solves its problems, and adapts itself to a world of impending disasters.

We have said that thinking, logically considered, is the process of making inferences, of arriving at knowledge. It is the process of passing from ideas already accepted to ideas which are inferred. The view has also been put forth that the psychological characteristic of logical thought, namely, controlled association of ideas, is a result of the fact that the thinker has a problem, an adequate solution of which demands that thinking must be so guided as to keep in touch with a world of objective realities. This, then, gives us the answer to a question which

is sometimes asked: *Why think?* Our reply is that we think because we are faced with problems which we have to solve.

But, someone asks, why do we have to solve problems? Is a problem something which is forced upon us from without—like the air we breathe—so that we are compelled to act with reference to the problem whether we wish to or not? On the contrary, is it not true that problems have a subjective origin? Are they not superimposed upon the environment from within? Don't we create our problems, instead of their being forced upon us from without? To answer these questions we must get down to the very fundamentals of the thinking process. We must undertake an analysis which will provide us with sufficient information so that we can define the nature of a problem. It is a strange fact that while human beings frequently discuss particular "problems"—social problems, problems of physics, philosophical problems, and so on—no one seems to be eager to raise the previous question of when we can be sure that we have a problem.

According to the outline which we will follow in the present volume, a *complete act of thought* passes through the following steps, or stages, the whole process being called a *cycle of thinking* because the completion of the final step modifies the original outlook (mental set) which made the problem possible in the first place:[3]

THE STAGES IN A COMPLETE ACT OF THOUGHT

I. *Origin of the Problem:* A result of the observation of some situation which arouses the investigatory response.

II. *Rise of Suggestion:* The emergence of an hypothesis (or induction).

[3] These steps in the complete act of thought do not coincide exactly with the steps outlined by John Dewey in his book, *How We Think.* However, the person who is interested in doing so can readily fit the two schemes, Dewey's and the present one, together.

III. *Deduction:* Inferring what other facts should be capable of discovery (observation), if the hypothesis be true.

IV. *Observation:* Looking for the facts predicted by the hypothesis.

V. *Verification:* The solution of the problem, through the finding of the facts which validate the hypothesis.

Before going further let us discuss each of these steps briefly.

I. Biologically a problem arises when we face a situation to which we must respond, but for which we have no ready made response. Some fact of the environment arouses our interest: we perceive a stimulus; perhaps the stimulus is ambiguous. At any rate we do not know just how to adjust ourselves to this situation which has some elements of novelty. There is aroused in us what may be called the *what is it?* reaction.[4] The problem of interpreting the situation and of reacting to it in some manner is thus forced upon us from the environment. The first stage in an act of thought is therefore the appearance of a problem; this sets off the thinking process. The goal of the thinking process is the solution of the problem, and this brings us to the next step in the act of thought.

II. The second step, according to our outline, consists in the rise of suggestions. A suggestion is a tentative clue, a possible solution to the mystery, or problem. A clue is usually obtained by looking into our past experience to find some situation analogous to the present one. In so far as an old situation is similar to a new one to that extent the old solution, or explanation, fits the new problem. But this explanation, or possible cause, is tentatively held, and therefore needs to be tested. Since it promises to bind the facts together, this suggestion may be called

[4] This label is used by I. Pavlov in his book on *Conditioned Reflexes.* Pavlov believes that the *investigatory reflex* is as definite a reflex as any other.

either an *hypothesis,* or an *induction.* However, the term "hypothesis" is usually taken to refer to tentative explanation, rather than to "laws" which stand in need of explanation, so we cannot identify all inductions with hypotheses. But, as we will point out later in much more detail, we are nevertheless justified in calling our second step one of induction.

III. In the third stage we deal with the means whereby the hypothesis, or explanation, is to be tested. This consists in making deductions from the possible solution. By way of illustration, let us assume that a murder has been committed. The problem immediately arises out of the observation of the facts: a man has been killed and the various facts point to the probability that this is not a case of suicide. The problem, then, is this: Who did it? And how was it done? Observation of a few facts (the most obvious ones) suggests the method employed in committing the crime. To test this hypothesis we look for the additional facts which ought to be capable of being discovered if this is the correct explanation of the method employed.

IV. The fourth step is that of looking for the facts predicted by the hypothesis. This, again, is observation, but it will be noted that this observation is no longer random and aimless, (as in the first step); it is directed in a definite direction. You are looking for specific facts. If you discover them this adds confirmation to your hypothesis. If everything points to the probable truth of the hypothesis, the supposed solution to the problem, the *hypothesis* ceases to be such and becomes a *theory.*

V. The last step of the complete act of thought is verification, the goal towards which we are working. If the predicted facts fail to appear we must then either modify or abandon completely our original hypothesis and formulate a new one. Always we must keep in touch

with the world of facts. As Herbert Spencer once remarked, many a beautiful theory has been killed by a nasty little fact. This is the impending tragedy of all scientific theorizing. But occasionally men do succeed in achieving a satisfactory solution to their problems, and when this happens that terminates the particular act of thought.

It will be observed that according to this analysis of thinking there is a very close relation between induction and deduction. The two play into each other, and are used as checks upon each other. A complete act of thought includes both these movements. It will also be noted that the whole process is cyclical. That is to say, when we finally verify an hypothesis, or solve a problem, this reorganizes our point of view. From that time forward we no longer look at the world in exactly the same way. In the future our attitude towards our environment, particularly that part of it in which this problem has arisen, will not be what it was previous to the solution of the problem. And this leads us back again to our original question of what a problem is.

What Is a Problem?

When we considered the nature of a problem the question was raised of whether a problem is forced upon us from without or whether we don't in a measure create the problems with which we are concerned. Our answer was that a problem originates in the observation of some fact (or set of facts) which calls for explanation. A problem may be considered to be something which interferes with the normal, habitual life-processes. The smoothly flowing currents of life are interrupted; at this point the organism adopts the *who goes there?* attitude. But—and here is an important point—*whether any given fact of the environment arouses in us the investigatory re-*

sponse depends on our interests, our training, our habits of thought.

Two persons may be in the same physical environment, but something in that environment attracts the attention of one person and fails to attract the attention of the other. The one individual sees a problem where another individual fails to see this same problem. A problem, apparently, cannot be explained solely in terms of a problematical environment. *A problem arises as a result of the relation between an environment and an interested organism.* And this observation brings us to one of the paradoxical things about thinking. We think because we have problems. Our aim is to solve the problem; that is, to get rid of the problem. In other words, *the aim of thinking seems to be that of removing the necessity for thought.* This suggests that human beings don't think any more than they absolutely have to, so that if thinking were not forced upon them by an unkind environment they would cease to think. But obviously this is false—when applied to some individuals at least. In fact, we may point out, one of the best symptoms of genius is the ability of an individual to *see* problems where other people fail to see problems. Sir Isaac Newton was a scientific genius because he could see the problem of gravitation, and—still more important—found the clue to the solution of that problem.

We must conclude, therefore, that a biological account of the origin and necessity for thinking is inadequate. And this is indeed the case. Originally, thinking served a utilitarian interest: Its function was that of enabling the organism to adapt itself to changes in the environment and to get on in the world, as we say. *But thinking, which was originally a means to an end*—that of survival in the struggle for existence—*has become an end in itself.* Thinking for some people has become a pleasure in-

dulged in for its own sake. Hence the statement that if we solved all our problems we would stop thinking ignores the fact that we sometimes create problems for the sake of solving them. If we had no problems we would set out in search for them, merely in order to give expression to this inclination to play with ideas. Human beings will never find themselves in the situation of Alexander the Great, who, after having conquered the world, sat down and wept for new worlds to conquer. Just as in science research is an "autocatalytic," or self-activating, process, so thinking in general always opens up new countries of the mind which invite further exploration.

The function of thinking is not to remove the necessity for thought, but to transfigure the character of our problems so that we substitute intellectual problems—mental experiments—for the more material problems of merely getting a living, finding food and shelter. We need not fear that the time will ever come when human beings will lack problems to occupy their minds. *Whatever an ideal society may be, it is certainly not one where intellectual idleness and stagnation have triumphed.* If we define a problem as a situation in which we want to get from here to there (where "here" and "there" may be defined in either a physical or a logical sense), then an ideal society would be one where the means of transportation is sufficient to carry us across the intervening logical or physical distance, while yet challenging us to exert our best efforts to catch the train of thought or action.

Since the entire next section is given over to an analysis of the organization of our knowledge, it will be well for us to summarize the foregoing views in so far as they are an anticipation of the subsequent exposition. What we have said is this: Mature organisms, or individuals with some background of experience, look out upon the world

with a certain mental set. This outlook is a result of a man's native abilities or natural predispositions as they have been molded by training and education. *What we see and the way we see it depend upon the kind of person we are,* and this is the resultant of our inheritance as modified by environmental influences.

In the above passage we have started with mature organisms. But nature does not start with mature organisms. How, then, did this mental set get built up? How does knowledge get started? How early in life can an individual have problems? When did he first begin to organize his knowledge? These are very difficult questions to answer. No matter where we start, we will probably find that a mental set is already on the road to being organized. We cannot discuss with very young children such questions as to when they were first conscious, had problems, and organized knowledge. We can only infer from their behavior what their inner life must be like. And the inferences about the mental life of children which various psychologists have drawn do not always agree. William James said that the world of a baby was a "big, booming, buzzing confusion," and many psychologists, following James, have tried to show how intellectual order comes out of the aboriginal chaos. But the gestalt psychologists disagree with this view. They tell us that experience is orderly from the very first; there are no meaningless experiences. A child, they say, does not start with pure, unmeaning sensations, from which chaotic manifold he then proceeds to build up an orderly world. A child reacts first of all to wholes—such as a "friendly" face—and within this world of patterns the parts are then more clearly and definitely articulated. This view, which has been expounded by Professor Koffka in his work on *The Growth of the Mind,* seems to exclude a definitely *pre-logical* stage (to

use Lévy-Bruhl's term), either in the life history of the individual or the evolution of the race.

However this may be, in a mature organism this mental set is composed of, and determined by, a loosely or closely knit (depending on the individual) body of beliefs, ideas, emotions, sentiments, and the like. It is this organized background which fixes a man's interests. Whether, in any given environment, a certain possible stimulus to thought will elicit our attention depends on this mental set, this apperceptive synthesis. Many people are indifferent to types of stimuli which present problems of a scientific or philosophic nature. Or better still, while most persons possess some measure of intellectual curiosity, they are unwilling to subject themselves to the long training and severe mental discipline necessary to build up that background which is required if one is to answer questions which result from this interaction of intellectual curiosity and the objective environment, the world of nature.

But all of us, scientists and laymen alike, do have some background of knowledge and experience, and to exhibit how this can be improved and most effectively employed is the purpose of logic. Deductive logic is the method by means of which we draw out the implications of the knowledge we already possess. Inductive logic is the technique we employ to construct new knowledge. While both of these interlock, and are used as checks upon each other in the progressive refashioning of the human mind, we clearly cannot discuss both of these at the same time. We must therefore break up our discussion of these two, and we will consider first of all the manner in which our already achieved knowledge is organized. After that we will turn to the logic of inductive science.

Chapter II

JUDGMENTS, PROPOSITIONS, AND MEANING

THE physiologist speaks of the structures of organs and the functions of organs. In the same way, the analyst of reasoning can break up thinking into two aspects, the structural and the functional aspects. Such a distinction is all the more necessary, since it turns out that the functional units of thought are not the same as the structural units.

What, then, are the functional and structural units of thought? The most elementary functional unit is *judgment.* Thinking is a process of combining judgments in a logical order to produce the conclusions that these judgments imply. Thus if you say, "The day is hot," you make a judgment, and there is nothing which can be psychologically simpler than such a process and still be the starting point for a reasoning process. If, after having made the first judgment, you reflect that on hot days organisms are apt to be thirsty you might draw the conclusion that your chickens at home probably need water. Or the judgment that heat causes evaporation might suggest that the electric battery in your car may be running dry. As we see in these cases, if we combine several judgments to draw a conclusion (make an inference) we are reasoning. Actual reasoning, therefore, is a complication of judgments related in a consistent fashion.

When a judgment is expressed in words it becomes a proposition, and propositions, like their subjective correlates, may also be called functional units of thought. Judgments may be private, but propositions, as written

or spoken judgments, are public property; and it is for that reason that we will talk about propositions rather than judgments.

Just as the reflex arc, as a functional unit of response, may be broken up into its structural units (sensory cells, connecting neurones, and motor cells), so the functional unit of thought, the proposition, may be broken up into its structural units. These are *terms* and *relations*. The process of reflection, implicitly expressed in judgment and explicitly stated as a proposition, is a process of relating terms to each other. All thinking can be reduced to the form of terms and relations,[1] which are logically and psychologically so simple that some authorities have not attempted to define them. For our purposes, however, a term may be defined as anything to which attention may be directed, which can be isolated from its environment or context and made the object of thought. Every term is a symbol which "stands for" some entity (logical or physical). Every term has some relation to every other term; the very assertion, "This has no relation to that," itself implies a relation between "this" and "that." A relation, then, is that which holds between terms. Where we are dealing with what are called correlative terms (such as teacher and pupil, cause and effect, etc.) it is hard to avoid defining one member independently of the other, and this is the case in connection with "terms" and "relations." They get their meanings from each other.

[1] What is called the *propositional function* is not in the term-relation form as it stands. The propositional function, represented symbolically in the form ϕ (x), is analogous to the mathematical concept of functional relationship. In propositions of ordinary logic the subject terms and the predicate terms are usually class names, and in symbolic logic a class consists of the terms that satisfy the propositional function. The propositional function with one variable is called a concept.

Propositional functions are discussed in C. J. Keyser's *Mathematical Philosophy,* Whitehead's and Russell's *Principia Mathematica,* and in Couturat's article in the *Encyclopedia of the Philosophical Sciences,* Vol. I, p. 157.

There are several types of terms and a number of different types of relations. We will discuss the different types of relations in a later chapter, but it may be remarked at this time that the relation we will be most concerned with is the relation of *implication*. Whenever the symbol $<$ occurs it refers to the relation of implication or inclusion. If a line is drawn through it thus $\nless$, this means the negation (or denial) of that relation. Thus $A \nless B$, means that A does not imply (or is not included in) B. In the propositions we will be most concerned with the subject term class is usually included in, or excluded from, the predicate term class. Thus when we say, "All men are mortal," or "No minors are voters," we are doing just that. While we are explaining our symbolism, we may also point out that, when not otherwise specified, the symbol "T" stands for term and "R" for relation; "S" stands for the subject term and "P" for the predicate term. It must always be kept in mind that the possibility for any assertion whatsoever rests upon the law of identity, $T \equiv T$.

The simplest form of thinking is represented by what is called *primary propositions*. Symbolically this is represented in this manner: $T_1R_1T_2$. Primary propositions assert a relation between classes. As a concrete example of this we have: "All men are vertebrates." But thinking is frequently more complex than this. A *secondary proposition* is one which asserts something about a primary proposition. As an illustration we have: "All men are vertebrates is true." This is represented symbolically thus: $(T_1R_1T_2)\ R_2T_3$. Assertions of even higher type are quite common. As an illustration of a *tertiary proposition* we have: "All men are vertebrates implies that no invertebrates are men." Symbolically this is represented as follows: $(T_1R_1T_2)\ R_2\ (T_3R_3T_4)$. Logically there is no limit to the number

of *types of propositions,* but psychologically thought is limited by the span of attention—the number of things we can talk about and still remember what we are trying to say.

It is interesting to note that many propositions which might ordinarily be thought of as belonging to the *first type* are frequently *secondary propositions.* When we say, for example, "The center of the earth is made of iron," we really mean this: "The center of the earth is made of iron *is true,*" or "*I believe* that the center of the earth is made of iron." This distinction is useful if we wish to avoid certain mistakes and paradoxes which may otherwise result. But to develop this idea we must turn to the problem of symbolism and meaning.

In the course of their investigations logicians have invented or discovered some paradoxes: these are contradictions which result from pursuing certain arguments. Whitehead and Russell argue that the solution of these paradoxes requires what they call the *theory of types.* We have touched upon this in the foregoing paragraph; now let us see how this theory is applied in concrete cases.

One of the famous statements attributed alike to Solomon and to Epimenides is that "all men are liars." It has frequently been pointed out that if this is so Solomon was also a liar; therefore what Solomon said was not true; therefore all men are not liars. Common sense can abolish this contradiction without resorting to the intricacies of the theory of meaning. It will be pointed out that what Solomon meant was that all men are liars in the sense that there is no man who has not told a falsehood at some time or other in his life. But this does not exclude the possibility that men also tell the truth sometimes, and that this is an occasion when Solomon was telling the truth. Thus the paradox vanished.

But now let us consider the following proposition:

"There is no truth." Common sense will have more difficulty in disposing of this paradox. If there is no truth then that proposition is itself false. But if it is false that there is no truth, then it must be true that there is truth; hence the original proposition is false again—and so on. The same contradiction occurs if a man says, "I am lying." What is the way out of these paradoxes? Apparently when men make these statements they do not intend that what they are saying should be applied to the statements themselves. Evidently some propositions do not apply to themselves; if a man is to speak significantly these propositions may be applied to anything else but themselves.

We see, therefore, that all propositions are not on the same level logically. Propositions may be of different types or orders. Part of the theory of logical types states that for any predicate there is only one type of subject to which the predicate meaningfully applies. According to this view there are sentences which, while grammatically correct, are literally nonsense. The contradiction is removed by denying that a meaningful statement is being made.

This distinction between logical types, primary, secondary, tertiary propositions, etc., has several important applications. For one thing it calls our attention to the fact that in order to contradict a given proposition the negative of the original proposition must be of the same logical type. Thus if we say, "The earth is round," the contradictory of that is "The earth is not round." These are both primary propositions. Now let us notice that there is no contradiction when Voliva says, "The earth is not round" and a scientist says, "I believe that the earth is round." The failure to see just where the contradiction occurs leads to confusion and argument. Russell believes that this is what happens when we get into prob-

lems concerning what really happens when we say, "All men are liars."

The argument here is somewhat slippery, but Russell's disposal of the paradoxes, "I am lying" and "There is no truth," amounts to this: propositions of the first type are statements about concepts or classes of objects, *and not about propositions*. At best you cannot make a statement about the totality of propositions in a proposition of the first type; you can only make a statement about the totality of individuals. As Russell[2] says: "If you are going to say something about all 'propositions,' you will have to define propositions, first of all, in some such way as to exclude those that refer to all the propositions of the sort already defined. It follows that the word 'proposition,' in the sense in which we ordinarily use it, is a meaningless one, and that we have got to divide propositions up into sets and can make statements about all propositions in a given set, but those propositions will not themselves be members of that set. For instance, I may say, 'All atomic propositions are either true or false,' but that in itself will not be an atomic proposition. If you try to say, 'All propositions are either true or false,' without qualification, you are uttering nonsense, because if it were not nonsense it would have to be itself a proposition and one of those included in its own scope, and therefore the law of excluded middle as enunciated just now is meaningless noise. You have to cut up propositions into different types, and you can start with atomic propositions or, if you like, you can start with those propositions that do not refer to sets of propositions at all. Then you will take next those that refer to sets of propositions of that sort that you had first. Those that refer to sets of propositions of the first type, you may call second type, and so on.

[2] *The Monist*, Vol. 29 (1919), p. 357.

"If you apply that to the person who says, 'I am lying,' you will find that the contradiction has disappeared, because he will have to say what type of liar he is. . . ." Thus when we specify the order of falsehood, or what comes to the same thing, the order of proposition to which falsehood is ascribed, no contradiction is involved.[3]

This doctrine that the attempt to apply propositions to themselves leads to nonsense can also be applied to the solution of certain paradoxes about words. Another of the famous paradoxes of logic is found in the problem, Is the class of all classes a member of itself (i.e., Is it a class?) or not? To escape the paradox—for, as examination will show, there are reasons why it must be and why it cannot be—Russell declares that the notion of "class" no more applies to itself than a proposition does. The fallacy consists in the formation of "impure" classes, i.e., classes which are not pure as to type. As Russell[4] says, to construct classes whose members are not all of the same grade in the logical hierarchy is to use symbols in a way which makes them no longer symbolize anything.

It must be pointed out, however, that some other investigators are not entirely convinced that these puzzles have been forever abolished by this device. It is held[5] that the theory of types is not as significant as it was originally thought to be. Not all the questions it was originally designed to answer require it, and in other cases there are better solutions to these paradoxes.[6] Ap-

[3] See also *Principia Mathematica,* Vol. I, pp. 39-68.
[4] *Introduction to Mathematical Philosophy,* pp. 136-7.
[5] Cf. Harry Weiss, *Mind* (1928), Vol. 37, pp. 338-348.
[6] Professor Henry Bradford Smith has argued that the paradox of Epimenides and Zeno's puzzles of motion can be disposed of through the development of an adequate theory of meaning. His views on this subject, which are in part a further development of the ideas of Professor E. A. Singer, Jr., are expounded in his paper on "Meaning" in the *Journal of Philosophy,* Vol. 26 (1929), pp. 182-185, and in other essays to which he there makes reference.

parently additional work on the theory of meaning needs to be done. It is especially to be desired that psychologists and logicians get together and clarify the problems along the borderline, that "twilight zone" where the two fields overlap.

Chapter III

TERMS, DEFINITIONS, AND CONCEPTS

IN the previous chapter we have seen that terms and relations are the structural units of thought. In the present chapter we must consider in more detail the nature, function, and kinds of terms. Since there are various types of terms it is advisable to discuss these classes of terms before entering upon a theoretical consideration of the uses and abuses of terms.

The several kinds of terms may be classifield as follows:

I. Singular, General, and Collective Terms
II. Concrete and Abstract Terms
III. Positive and Negative Terms
IV. Absolute, Relative, and Correlative Terms

Let us take these up in the order named.

I. A *singular* term is one which applies to a single individual only. If we speak of the Niagara Falls or the Torch in the Hand of the Statue of Liberty we are using singular terms. It will be observed that while these terms consist of a number of words, they nevertheless are single (as well as singular) terms, because they refer to objects of single acts of attention. Proper names, such as the term Abraham Lincoln, are also singular terms. But such terms as a Judas, or a Mephistopheles, are general terms.

A *general* term is one which applies to an indefinite number of objects. Such class names as: metals, book, soldier, etc., are general terms. Such terms are distributive over the whole group to which the term applies.

A *collective* term applies to a number of individuals taken as a group. As illustrations we have: audience, mob, regiment, and the like. Any individual who is properly qualified may be called a soldier. Soldiers are combined to form companies, regiments, and armies. But the terms "army," "regiment," "democracy," and the like cannot be applied to the individuals who make up these groups. It has been suggested that the term "mind" ought to be thought of as analogous to the term "democracy," as expressing a mode of togetherness.

These distinctions must be kept in mind if confusion is to be avoided. The term "all," for example, is ambiguous; on some occasions it is apt to be regarded as a general term, and on other occasions it may be used as a collective term. Thus if we say, "All the angles of a triangle are less than two right angles," *all* here means "each and every." But if you say, "All the angles of a triangle are equal to two right angles," *all* here means "all taken together." Unless care is taken other ambiguities will result.

II. A *concrete* term is one which refers to some object of sense perception, a *particular*. The term "a green tree" is a concrete term, because it refers to an object which can be perceived. Since it is usually easy to find the object to which the term refers these terms are not likely to cause trouble. But abstract terms no longer refer directly to particulars, and they therefore easily lead to difficulties.

An *abstract* term refers to an object of thought which is a product of an act of abstraction. This gives us the *universals,* which, as results of ideation, cannot be perceived through the senses. Abstraction is the process of drawing out from a group or a class of objects the common qualities, isolating them, and referring to them through the use of universals, such as greenness, justice,

charity, tallness, and so on. Abstract terms are sometimes far removed from their original concrete particulars, and therefore are hard to define. Plato and Socrates spent a good part of their lives trying to discover what "goodness" and "justice" are. And modern physicists are trying equally hard to find out what "energy" is. All these are abstractions.

III. *Positive* and *negative* terms are *contradictory* terms. That is to say, there is no middle ground between them. A positive term indicates the presence of some positive quality. Such terms as "good," "hot," "sweet," and so forth, are positive terms. The true negative of a term indicates the absence of this quality, though not necessarily the presence of the opposite quality. Thus the contradictory of moral is non-moral, and not immoral. If one wants to cover the whole ground with respect to some division into classes he should use the original quality and its negative as obtained by prefixing *non* or *not* to it. Thus the classification of men into fools and knaves does not cover the whole field of human beings, whereas the dichotomy into fools and not-fools does. This mode of division of terms employs the law of excluded middle: that A is either B or it is not-B. The opposition between hot and not-hot is one of *contradiction,* but the opposition between hot and cold (which admits of a middle ground, such as warm) is one of *contrary* terms. This distinction, which is not so essential at this point, will be of use later when we consider contrary and contradictory propositions.

IV. In connection with the discussion, in the preceding chapter, of terms and relations we have noted the fact that some terms are *correlative* terms. Correlative terms are a special case of relative terms. Relative terms get their meanings through their relations to something else. Thus the terms ignorance, tolerance, and the like,

are relative. When relative terms are paired off the members of the correlative terms are called *correlates*. Later we shall consider the way in which the terms *cause* and *effect* afford illustrations of correlative terms.

An *absolute* term, in contrast with a relative term, is one which has a comprehensible meaning when considered alone. Thus the terms cat, tree, the United States, etc., are illustrations of absolute terms.

Words change their meanings with the processes of time. If the earlier meaning of the term lingers on alongside the new meaning the word becomes ambiguous. Much confusion results from the fact that words may have several meanings. The word *law,* for example, has several meanings. There is the meaning it has when we speak of civil law. But the term is also used in natural science. This ambiguity has led to the type of argument which is illustrated by the following:

> All law implies a law-giver;
> Nature has law;
> Therefore, nature has a Law-giver.

This argument has been accepted by some theologians and philosophers as a valid proof for the existence of God, who is then conceived to be the maker of natural laws. But every scientist knows that a law of nature is not comparable to a law enacted by a legislative body.

In the above case the word law has changed its meaning by a process of *generalization.* In such cases a word comes to have a wider meaning than it originally had. But words may also change meaning by the process of *specialization.* Thus the term minister originally meant anyone who served, but now it is specialized to mean a clergyman or a representative of the state. Words may also get their meaning as a result of association (by similarity, usually), so that, for example, we now speak of the "hands" of a clock or the "feet" of a table.

The device which is commonly employed to fix the meaning of a term is *definition.* Definition is the process by means of which we assign a precise meaning to the terms we use. Ordinarily we define the meaning of a term by giving the essential qualities and attributes (the *connotation*) of the word to be defined. Thus we would define the term *life* in a biological sense by incorporating in the set of words which we substitute for the term to be defined the characteristics of irritability, metabolism, growth, reproduction, etc. In other cases we may resort to defining by giving the *genetic formula.* Thus, for technical purposes, sulphuric acid might be defined by giving the structural formula: H_2SO_4. Following the same principle, one might suppose that life could also be defined in terms of the formula of the protoplasm which is its physical basis. But this supposition calls to our attention a defect of this type of definition. In every case where the whole to be defined is something different from the sum of the properties of the parts, a structural or genetic formula does not tell us much about the whole as such. Thus it does not help us much in understanding the meaning of the term water to be told that it is H_2O. The *fallacy of over-analysis* consists in identifying a thing with its conditions or elementary constituents. We will return to this matter when we consider the nature of *explanation.*

But no matter how one defines a thing, in our definition certain requirements ought to be met if our definition is to be a good one. These requirements are as follows: (*a*) The definition should state the essential qualities and attributes of the thing to be defined; (*b*) the definition should not be stated in figurative or obscure language; (*c*) the definition should neither be too broad nor too narrow; and (*d*) the definition should be stated in the affirmative whenever this is possible. Violations of these

rules are quite common and the student will do well to be critical about these matters.

This completes what we have to say about the commonly accepted body of information which is discussed in text-books in logic in connection with the subject of definition. The unfortunate thing is that many texts stop at the point where the real problems begin. It is apparently supposed that the more "philosophical" problems ought to be reserved for some more advanced treatment, such as is presented in a course in the philosophy of science. This is surely a mistake, for of what benefit is this sharpening of the tools of thought if no use is made of these tools along the cutting edge, the thorny frontiers, of human knowledge?

Let us therefore plunge into this wilderness of verbalism and linguistic underbrush and see whether we cannot cut our way through into the open again. We must again begin with words in their symbolic aspects.

Concepts and Symbolism

It is a commonplace of human observation that most intellectual disputes ultimately resolve themselves into differences of opinion concerning the meanings of words. Language, which ought to be the servant of thought, sometimes usurps the throne of reason and tyrannizes over men who then become the slaves of words. The domain of thought ought to be a democracy, where words express the intentions of ideas, but in actual fact an autocracy of verbalism is frequently being practised. In such a situation, as G. K. Chesterton once pointed out, men cannot mean what they say because they do not say what they mean.

As we have previously pointed out, this situation has come about as a result of the fact that man lives in two worlds: a world of objects and a world of words. It is as

hard for man to live in these two worlds as it is for an equestrian to ride two wild horses at the same time. If, on the one hand, man lived only in a world of physical objects, and, like other animals, was content merely to survive and adjust himself to other objects he would not get into the intellectual difficulties which confront him. Or if, on the other hand, man lived entirely in an ideal world, the domain of abstract concepts, and faced no necessity of adjusting himself to the world of brute facts, he would likewise suffer no conflicts. In the first case man would have no intellectual problems of the type we have in mind because he would have no conceptual abstractions to relate to a world of empirical reality. In the second case he would be undisturbed by these problems because there would be for him no world of objective facts and relations to which he would have to relate his ideational constructs.

A concrete example of the type of problem we have in mind may be found in physics. To develop this illustration we will first have to recall the distinction, made in an earlier chapter, between mathematics and physics. We saw that the content of both mathematics and physics takes the form of the statement *p implies q*. The difference between mathematics and physics was stated to consist in the fact that in mathematics the properties of the "*p*'s" and "*q*'s" are determined by postulation, whereas in physics they are determined by inspection. This distinction, which is in accordance with common usage, is brought out clearly by A. S. Eddington[1] in the following passage:

> The pure mathematician deals with ideal quantities defined as having the properties which he deliberately assigns to them. But in an experimental science we have to discover properties, not to assign them; and physical qualities are defined primarily according

[1] *The Mathematical Theory of Relativity*, p. 1.

to the way in which we recognize them when confronted by them in our observation of the world about us.

This mode of procedure is in theory entirely commendable. The only trouble is that in many actual cases it is hard to draw a clear line of demarcation between *definition by postulation* and *definition by inspection.* For, as is known, science necessarily deals with abstractions, which already contain ideal elements in them, so that it is difficult to tell where the subjective elements (associated with the conceptual process) leave off and the objective conditions begin. One of the best illustrations of this—and here we come to our specific example—is presented by Professor P. W. Bridgman.[2] After quoting Newton's statement that "Absolute, True and Mathematical Time, of itself, and from its own nature flows equably and without regard to anything external, and by another name is called Duration," Professor Bridgman urges that "there is no assurance whatever that there exists in nature anything with properties like those assumed in the definition."

The difficulty of distinguishing between what things really are and what we take them to be is of course greatly increased when we come to consider strictly psychological (subjective) phenomena. By way of illustration, let us consider the concept of mind or soul (for us the two will be synonymous). Assuming that the soul exists in some sense, how shall we define it? Various definitions have been proposed. According to popular belief, which is interested in a conception making immortality possible, the soul is a non-material entity or principle which somehow acts on the body, though it can exist in separation from the body, as after death.

[2] In his book, *The Logic of Modern Physics* (p. 4).

Scientists have found this concept of the soul unsatisfactory, for reasons which are so evident as not to need statement. Other definitions might present fewer difficulties from the viewpoint of science. Among such there is the concept which defines the soul as the *harmony of the body*. According to this view, which is as old as Pythagoras, what musical harmony is to the instrument which produces it the soul is to its own body. This definition, which was rejected by Plato, is not so very different from Aristotle's definition of the soul as the *form* of the body. These latter definitions of the soul involve no violation of physical principles and therefore might be acceptable to a modern behaviorist, as several writers have pointed out.

And now the question arises, Do we have the right to make our definitions to suit ourselves? Can we make words and concepts mean whatever we want them to mean? If we have the freedom to define the soul in any way we please the soul can be made to exist by definition, provided, of course, we find some empirical basis for our concept. This is what we have done when we define it as the harmony (or functional integration) of the body. In the same way, God's existence might be assured by defining Him as the personification of social values, such as love, justice, mercy, etc. God then exists in the same sense in which Uncle Sam or Santa Claus exist. The objection which some would make to these definitions is that the soul and God are not what we choose to make them. These terms, the devout will declare, refer to independent realities, and our definitions must therefore reveal their real natures as such. But that is the precise difficulty. What is the *real* nature of the soul and of God?

Now we must take up this problem of the relation of

the name to the thing signified by that name. This is a problem which the logician has an obligation to be interested in, and which he may hope to solve.

The name is frequently so closely linked with the concept which it is associated with that ordinarily we do not distinguish between the symbol and the symbolizing process. It is for this reason that we commonly suppose that if there is a name there must be a reality corresponding to the name. This is illustrated in the story of the lady who listened to an astronomer lecture on the stars and planets. After the lecture the lady informed the Professor that she enjoyed his talk *so* much, and understood everything, except one little point: How did the astronomers ever discover the names of the stars and the planets? The answer to this question is so obvious that we needn't bother to state it. If astronomers had to discover the names of the planets they would be as bad off as the little dog in the railroad station whose destination was uncertain because he had eaten off all his labels.

This confusion of the name, the thing named, and the naming process is so common that we need to consider the matter further. The point which should be emphasized is that the names, or symbols, are purely arbitrary: we might, for example, have called "black" "white" and "white" "black" and we would be no worse off than we are now. Kunckel, the discoverer of phosphorus, was much disgusted with the practise of the alchemists of giving different names to the same substance. He pointed out that while one is perfectly free to baptize his infant as he pleases, calling an ass an ox will never make one believe that an ass is an ox. But this does not controvert the statement that the name is optional; the only thing to be remembered is that if we want to avoid confusion we should not call two different objects by the same name or use the same name to apply to two different objects.

This point is well made by Humpty-Dumpty in Lewis Carroll's *Through the Looking Glass:*

"When *I* use a word," Humpty-Dumpty said in a rather scornful tone, "it means just what I choose it to mean—neither more nor less."

"The question is," said Alice, "whether you *can* make words mean so many different things."

"The question is," said Humpty-Dumpty, "which is to be the master—that's all."

If Humpty-Dumpty meant by this that he had the right to attach a word to whatever object he pleased he was correct in his statement. But if he meant that by transferring a name from one object to a second object this object then became identical with the original object, he was merely falling victim to primitive magic and sorcery. Such magic has occasionally been resorted to even in science.

As Dr. H. M. Johnson[3] has pointed out, psychology today is prone to fall into this linguistic trap. This is probably the case in all young sciences. In this respect the psychology of today is in the position of chemistry several hundred years ago.

The early modern chemists, who were struggling to free their science from the mysticism of alchemistic doctrines, had to fight their way through the word magic which the modern psychologists are now wading through. Robert Boyle was one of the first to launch an attack upon "occult qualities," that sanctuary of the ignorant, as he called them. Boyle had no higher opinion of the chemical knowledge of his contemporaries than a modern behaviorist has for the methods and results of subjective psychology. In his book, *The Sceptical Chymist,* Boyle says, "Methinks the chymists in their search after truth

[3] "Some Fallacies Underlying the Use of Psychological 'Tests,'" *Psychological Review,* Vol. 35 (1925), pp. 328-337.

are not unlike the navigators of Solomon's Tarshish fleet, who brought home from their long and perilous voyages, not only gold and silver, and ivory, but apes and peacocks too: for so the writings of several (I shall not say all) of your hermetick philosophers present us, together with diverse substantial and noble experiments, theories, which either like peacocks' feathers make a great show, but are neither solid nor useful, or else like apes, if they have some appearance of being rational, are blemished with some absurdity or other, that, when they are attentively considered, make them appear ridiculous."

And now let us ask, What is the way out of this muddle? Before we attempt to answer that question let us first restate how we got into it.

Science, it is generally agreed, is concerned with measurement. In physics you measure weights, potentials, velocities, and the like; in biology you measure growth, elementary responses, energy turn-over in the organism, and so on; in psychology you attempt to measure memory, attention, learning, intelligence, fatigue, etc. The problem, we have seen, appears when you can no longer be sure (*a*) that there is something objective corresponding to the term which is supposed to symbolize the entity or process you are trying to measure, and (*b*) even though something does exist which corresponds to what the term is supposed to represent, whether this entity or process has the precise qualities or properties which are assigned to it. In brief, our difficulties arise as a result of our inability to distinguish between entities which are defined by postulation (i.e., non-observables) and induction (observables).

Let us next observe that if anything is to be done to remedy this situation, there are three possible modes of procedure open to us:

(A) We might remove the source of difficulty by

agreeing that all definitions must be arrived at through inspection, through inductions from particulars, and exclude from rigorous thought all hypothetical entities not directly observable. Some physicists of the past and of the present have advocated this method as the proper one for physical science. The psychological behaviorists would probably also approve of this procedure. They appear to agree that non-observables have no place in science, hence definition through inspection alone ought to be permitted. The behaviorists argue, for instance, that such terms as *sensation, attention, image,* and the rest, are not accessible to scientific observation and therefore ought to be excluded from the domain of scientific psychology. But at this point the introspectionist is likely to object. He may point out that while it is true that conscious processes are not accessible to social observation, nevertheless a conscious process can be pointed to. If you ask for a definition of the term consciousness the introspectionist may reply: "I mean by consciousness that which you experience when you ask that question." Here the introspectionist is *pointing to*—in a figurative sense, it is true—but that is all we can do when, for example, a person asks what is "sweetness"? You give the person a lump of sugar and tell him sweetness is what he experiences when he eats the sugar.

(B) The second possibility consists in formulating all definitions by postulation. There are reasons for believing that this is the method we usually employ, even in cases where we do not realize it. According to the *"as if"* philosophy all our concepts are fictions; at best they are useful devices which we superimpose upon external nature. Most of these subjective constructs have been standardized, so that as conventional artefacts they have great utility.[4]

[4] For two different versions of this viewpoint see H. Vaihinger's *The Philosophy of "As If,"* and V. Steffanson's *The Standardization of Error.*

That we actually employ this method may be seen in the fact that if, as a result of a few observations, we define a thing in a certain way and then come across something else which possesses some of the properties of this class of things but fails to meet the requirements in all details *we don't inspect it.* We say it isn't that kind of thing. Thus, if we define a crow as a black bird of a certain type and then come across a white crow (a new mutation, let us suppose) we may save our generalization that all crows are black by the statement that this cannot be a crow because a crow is black by definition. This brings up the problem of the objectivity of biological classes, but all we can say on this point is that it now seems pretty well established that species do not represent distinct types such as were supposed, by the "special creation" theory, to exist in the mind of the Creator.

(C) The third possibility is, of course, to continue the present practise of employing both types, definition by postulation and by induction. Mathematics certainly requires freedom to employ the first type of definition. And it is equally certain that the natural sciences must be restricted to the formulation of concepts and principles which approximate as closely as possible the empirical facts. In cases where *something* is objectively real the desirable thing is to make our intellectual constructs coincide as closely as possible with the things and events which are being studied.

Natural science has been built up by a "give and take" process. The definitions and hypotheses of empirical science must keep in touch with the world of itemized facts. Definitions are tentatively formed to fit preliminary observations, but as time goes on further observations reveal finer details which do not fit the formulæ, so the definitions or formulæ are modified accordingly. Thus by the reciprocity between the observed facts and

logical constructs systems of knowledge are slowly built up. Scientific systems do not possess the finished neatness and comprehensiveness of some philosophical systems, but what they lack in this respect they make up for in solidity and specificity.

Chapter IV

THE RELATIONS OF PROPOSITIONS

IN the previous chapter we were concerned with terms, the structural units of thought. Now we come to propositions, which, as overtly expressed judgments, constitute the functional units of thought.

Reasoning and argumentation are ordinarily presented through the medium of sentences. But not every sentence is a logical proposition. Grammarians enumerate various types of sentences: declarative, exclamatory, interrogatory, optative, imperative, and hortatory. But sentences which express a wish, which ask a question, which exclaim, or which incite, do not enter directly into the processes of proof, and are therefore of secondary importance from the point of view of logic. We see, therefore, that only sentences which are true or false are subject matter for logic.

But these (declarative) sentences must be put in logical form before they can be considered as logical propositions. Such sentences as "the earth moves" or "it rains," are not in logical form. Before they can be dealt with as the type of proposition which occurs in the syllogism they must be restated. A logical proposition states a relation between a subject and a predicate term. As already pointed out, the relation we will be most concerned with is the relation of class inclusion. This relation is usually indicated by the copula, some form of the verb "to be." Thus if we say, "All students pay tuition," we are including the subject term class, students, in the

class of the predicate term, persons who pay tuition. In its full logical form this is, "All students are persons who pay tuition."

This statement is called a *categorical* proposition. If it so pleases us, we may employ other types of propositions. A *conditional* proposition states a conclusion as dependent upon a prior condition; thus, "If it rains, we will not go." The use of this type of proposition leads to the *hypothetical* syllogism. But there are also *disjunctive* propositions, such as, "This object will either sink or swim." The combination of the hypothetical and disjunctive arguments is called the dilemma. The following is the customary classification of propositions and syllogisms:

Propositions (*Or, Syllogisms*) { *Categorical*; *Conditional* { *Hypothetical*; *Disjunctive* }

Now let us return to the simple categorical proposition. In the examples we have used the subject term class is included in the predicate term class. But we also make negative statements, and here the subject term class is excluded from the predicate term class. Thus we may say, "No minors are voters." This distinction between affirmative and negative propositions is referred to as the *quality* of a proposition. It is also obvious that we do not always talk about the whole of a given class. We may say, "*All* students pay tuition"; but we can also say, "*Some* students are industrious." This difference in the distribution of the terms is referred to as the *quantity* of a proposition. A proposition which affirms or denies something about the *entire* subject term class is called a *universal* proposition, while one which affirms or denies something about only a *part* of the subject term class is called a *particular* proposition. This sums up as follows:

$$\textit{Quality}\begin{cases}\textit{Affirmative}\\ \textit{Negative}\end{cases}\qquad \textit{Quantity}\begin{cases}\textit{Universal}\\ \textit{Particular}\end{cases}$$

It is readily seen that we have only four kinds of propositions to deal with. These are as follows:

$$\textit{Universal}\begin{cases}\textit{Affirmative} & \text{—}\textit{All } S<P \text{ an A proposition}\\ \textit{Negative} & \text{—}\textit{No } S<P \text{ an E proposition}\end{cases}$$

$$\textit{Particular}\begin{cases}\textit{Affirmative} & \text{—}\textit{Some } S<P \text{ an I proposition}\\ \textit{Negative} & \text{—}\textit{Some } S\not<P \text{ an O proposition}\end{cases}$$

The letters A and I come from the Latin term *affirmo*, while the letters E and O come from the Latin *nego*. As examples of the four types of propositions we have:

A—All criminals are dangerous to society
E—No planet is self-luminous
I—Some stars are double
O—Some citizens do not vote

There are several types of statements which are a little difficult to classify. First of all, there are so-called *indefinite* propositions. If someone says, "Men are to be trusted," one wonders whether this means *all* men or *most* men. But since no restriction on the distribution of the term is expressed, this is to be considered as a universal proposition. *Individual* propositions also deserve a word. Such statements as, "Socrates is mortal," or "Mars is inhabited," are considered to be *universal* propositions and not particular propositions. You are here talking about the whole class, but the class has only one member. Finally, so-called *exceptive* propositions must be mentioned. Propositions introduced (or qualified) by "only" and "none but" require more thought before they can be classified. What kinds of propositions are the following? "Only ticket holders will be ad-

mitted"; "None but the brave deserve the fair." To bring out the meanings of these propositions they need to be restated. The proposition, "Only ticket holders will be admitted," means that those who do not hold tickets will not be admitted; as thus stated, this is an E proposition. The statement, "None but the brave deserve the fair," means that those who are not brave do not deserve the fair. Here, again, we have an E proposition.

Now we must say something about the distribution of terms. The distribution of the subject term tells us whether the proposition is universal or particular. The sign of a universal proposition is the word *all,* or some synonym for it. The sign of a particular proposition is the word *some,* or some synonym for it: the range of some varies between "a few," "many," and "most"—but not all. If you are not talking about the whole of a class denoted by a term, the term which symbolizes the part of the class you are talking about is *undistributed.* Since every proposition has a predicate term as well as a subject term, each of these will have its distribution. The way in which the distribution of the subject term is determined we now know. We need only examine the distribution of the predicate term.

If, as in an A proposition, you say, "All men are vertebrates," you are clearly not talking about all vertebrates, but only those which are men. In all universal affirmative propositions the predicate term is undistributed. And the same reasoning applies to the I proposition, the particular affirmative, so that the predicate term of an I proposition is also undistributed. In brief, *in all affirmative propositions the predicate terms are undistributed. In the negative propositions, the E and the O, the predicate term is always distributed.* If you exclude either the whole (in the E) or the part (in the O proposition) of the subject term class from another class

(the predicate term class) you must necessarily have examined the whole of that class. These conclusions may be summed up in the following form:

Proposition	Distributed	Undistributed
A	Subject	Predicate
E	Subject Predicate	———
I	———	Subject Predicate
O	Predicate	Subject

Euler's Diagrams.—The meaning of propositions can be represented graphically by the use of circles, a practise first employed by the German mathematician Euler:

The A proposition: All S<P.....

The I proposition: Some S<P.....

The E proposition: No S<P.....

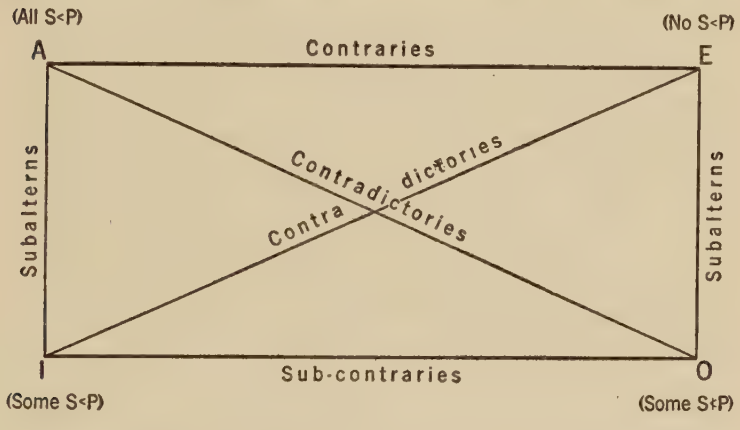
The O proposition: Some S≮P.....

Immediate Inference

In deductive logic two types of inference are recognized; these are *mediate* inference and *immediate* inference. Mediate inference is illustrated by syllogistic reasoning; it is inference through the *mediation* of a *middle* term. We will reserve our discussion of this until the next chapter. Now we are concerned only with immediate inference.

As the term immediate indicates, this type of inference is made directly and at once. Given the truth or falsity of a proposition one can tell immediately whether some

other related proposition is true, false, or indeterminate. Given any one of the four kinds of propositions, either the A, E, I, or O, one can restate it in any of the three remaining types, and know at once the truth or falsity or indeterminateness of the new proposition. This is summed up in what is called the square of oppositions:



In dealing with terms we employed the distinction between *contraries* and *contradictories*. The same distinction is useful in connection with propositions. Between contradictory propositions there is no middle ground: if one is true the other is false, and if one is false the other is true. But contraries and sub-contraries are not so easily disposed of. If one member of a pair of *contrary* propositions is true the other member of the pair is false, but, as examination will reveal, *both may be false*. The A proposition, "All pretty girls are intelligent," and the E proposition, "No pretty girls are intelligent," may both be false: the I proposition, "Some pretty girls are intelligent," and the O proposition, "Some pretty girls are not intelligent," express the real truth. In connection with *sub-contraries*, we must note that if one member of the pair is false the other is true, but *both may be true*. The

propositions, "Some students are industrious," and "Some students are not industrious," are sub-contraries. Both of these may be true; if either be false the other must be true; but only one need be true. It will also be observed that while sub-contrary propositions are consistent with each other, contradictory and contrary propositions are inconsistent.

In the case of subalterns, both are of the same quality, but they differ in quantity, the upper member of the pair being the universal while the lower is a particular proposition. If the universal is true the particular must also be true, but the truth of the particular does not guarantee the truth of the universal. On the other hand, the falsity of the universal does not guarantee the truth of the particular. In order to fix these relations in mind it is advisable for the student to fill in the following schematism:

If A is true, E is false,	I is true,	O is false
If E is true, ?	?	?
If I is true, ?	?	?
If O is true, ?	?	?

If A is false, E is indeterminate,	I is indeterminate,	O is true
If E is false, ?	?	?
If I is false, ?	?	?
If O is false, ?	?	?

The fact that people make so many mistakes in trying to see the relations between propositions indicates that we do need practise in this field. No great difficulty arises when the propositions considered have the same subject and predicate terms, as above, with variations only in the quantity and quality of the propositions. But when propositions which are logically related (i.e., in the same "universe of discourse"), though having different terms, are being considered, difficulties of a more imposing sort are encountered. Such statements as "Aristotle is the wisest of the Greeks" and "Socrates is the greatest

thinker of antiquity" are relatively simple illustrations. Both cannot be true; and both may be false. But in other cases the answer is not so obvious. What we aim at when we desire consistency is *freedom from contradiction.* This may be called the highest good (*summum bonum*) of logic. We must examine the anatomy of consistency in more detail in a separate chapter.[1] In the present chapter there still remains to be discussed, in connection with immediate inference, the processes known as the *conversion* and *obversion* of propositions.

Conversion.—It is frequently desirable, in order to exhibit the full meaning of a proposition, to turn it around, so that the original subject term is now the predicate term. This process is known as *conversion.* In the new proposition the distribution of the terms remains the same, the meaning of the proposition is unaltered, but the order of the terms is reversed. Thus, if the original proposition (called the *convertend,* the "to be converted") is an A proposition, its converse will have to be an I, in order to preserve the undistributed character of the original predicate term. Concretely, if it is true that all traitors are despisable, we can only conclude with certainty that *some* beings who are despisable are traitors. It is therefore said that the A proposition is converted *by limitation.* But an E or an I can be converted "simply," because in these cases the distribution of the subject and predicate terms remains the same.

It is essential to the process of conversion that the converse of any proposition should have the same quality as the original. Thus the converse of an affirmative proposition is an affirmative proposition, and the converse of a negative proposition is a negative proposition. But, among negatives, only the E proposition has a converse. The O proposition has an undistributed subject term in its original form; by conversion this subject term

[1] Chapter VII of this Part.

would become the predicate term of a negative proposition, thus becoming distributed—which is contrary to the rule that the distribution of the terms must remain the same. The following sums up the foregoing points:

The Convertend	*The Converse*
A—All S<P	I—Some P<S
E—No S<P	E—No P<S
I—Some S<P	I—Some P<S
O—Some S⊄P	No Converse

The student will do well to formulate some propositions and practise converting them.

Obversion.—It is a common practise for a writer or a speaker to state a conclusion in negative form. In such cases one is always interested in knowing what this means in positive form. Or it may be desirable to perform the opposite operation. Logical sequence or psychological emphasis will determine whether we put our propositions in negative or affirmative form. But every proposition can be restated as a proposition of opposite quality. This process of changing a proposition from negative to affirmative or from affirmative to negative is called *obversion.* While the quality of the proposition is changed, the meaning must remain the same.

To obvert an A proposition it is necessary to proceed on the principle that two negatives make an affirmative. As we have seen in discussing the law of contradiction, if A is A, A is not not-A. If man is a political animal, man is not a non-political (unsocial) animal. The forms for all the obverted propositions are as follows:

	The Obvertend	*The Obverse*
The obverse of A is E:	All S<P	No S<P′
The obverse of E is A:	No S<P	All S<P′
The obverse of I is O:	Some S<P	Some S⊄P′
The obverse of O is I:	Some S⊄P	Some S<P′

Contraposition.—Contraposition is the process of combining the two preceding processes. The contrapositive of a proposition is usually said to be the result of obversion plus conversion; though some logicians add a second obversion. But even this does not represent the limit of the combined operations of obversion and conversion. These processes can be continued until we arrive at what is called the *full inverse.* However, in actual practise we seldom find it useful to carry the processes of immediate inference any further than the simple, uncomplicated obversion or conversion of propositions. Nevertheless, it is excellent training in abstract thought to practise giving all the immediate inferences one can from the original propositions. The following[2] give abstract and concrete examples of this:

1. (Original Proposition) —A—All probable events are possible
2. (Obverse) —E—No probable events are impossible
3. (Converse) — I—Some possible events are probable
4. (Obverted Converse) —O—Some possible events are not improbable
5. (Partial Contrapositive) —E—No impossible events are probable
6. (Full Contrapositive) —A—All impossible events are improbable
7. (Partial Inverse) —O—Some improbable events are not possible
8. (Full Inverse) — I—Some improbable events are impossible

It is an interesting question to discuss how we know whether any given inference (immediate or mediate) is valid or invalid. Formalists will argue that the validity

[2] The mathematically minded reader should consult J. N. Keynes' *Studies and Exercises in Formal Logic* (4th ed., Chs. 3 and 4) or A. E. Avey's *The Function and Forms of Thought* (Ch. 16) for a fuller discussion of these processes of immediate inference.

of the processes of immediate inference does not depend upon appeal to experience. Valid inferences, they hold, may be *confirmed in* experience; but they are not *established by* experience. Here we again touch upon the nebulous borderline of logic and psychology. We cannot settle all these matters now, and will come back to some of these problems in the chapter after the next. In the meantime the following views advocated by Mrs. Ladd-Franklin throw some light upon this twilight zone of logical psychology or psychological logic.

According to Dr. Christine Ladd-Franklin,[3] there is a type of inference which is commonly employed which has not yet been treated in books on logic. An example is as follows: If I say, "Green cheese is indigestible," I do not, by means of *logic,* convey the inference that non-green cheese is not indigestible,—nevertheless, there is a fundamental pleasure-pain principle which does enable me to make this deduction. This principle is the psychological law of parsimony: no human being ever takes unnecessary trouble. Accordingly, if we had meant to say that cheese in general is indigestible we would not have taken the trouble to specify green cheese. It is therefore psychologically correct to make the inference, and the logically unsophisticated persons who habitually employ this type of inference have a genuine flair for what Mrs. Ladd-Franklin calls *instinct logic.*

[3] "The Law of Parsimony: Psychological Logic," *Journal of Philosophy,* Vol. 24 (1927), p. 476. See also her article on "The Antilogism," *Psyche,* October 1927, 100-103.

Chapter V

THE SYLLOGISM: ITS USES AND LIMITATIONS

DURING the period of Scholasticism the syllogism was highly prized as a type of argument. And quite naturally, too; for if—as was the case during Medieval times—the premises of an argument are already established (by revelation and authority), all that the thinker can do is draw out the implications of that which is assumed as certain or self-evident. Hence the exaltation of the syllogism by the learned men of those times.

But in modern times the syllogism has fallen somewhat from its former high estate. The appearance and spread of the experimental attitude and method are sometimes taken as complete repudiations of syllogistic reasoning. But this is certainly carrying the reaction too far. We must always remember that modern science, in so far as it is mathematical, uses the deductive method. And indeed, as has frequently been pointed out, a discipline is "exact" and scientific precisely to the extent that its principles and generalizations lend themselves to mathematical treatment and statement. This means, then, that science does not dispense altogether with deductive and syllogistic logic; but, rather, that it puts deductive inference in its proper place as a method for exhibiting the implications of the knowledge we already have. The Medievals, therefore, are not to be criticized for resorting to deductive logic, but for employing it almost exclusively and in isolation from inductive procedure.

We will first consider the nature and types of syllogistic reasoning, after which we will examine in more detail the values and limitations of this mode of argument.

Syllogistic inference is termed *mediate* inference, because the inference is *mediated* by the *middle* term. The typical syllogism is an argument with two premises and a conclusion. The premises give the grounds for the conclusion, and are the result either (*a*) of previous inductions, or (*b*) postulation, as in mathematical science. The middle term occurs in these two premises, but does not appear in the conclusion. The *predicate* term of the conclusion is known as the *major* term and the *subject* term of the conclusion is called the *minor* term. The premise which contains the major term is called the *major premise;* the premise containing the minor term is called the *minor premise.*

The syllogism, therefore, will contain three premises and three terms, the apparent exceptions to this not being real exceptions. In what are called *enthymemes* one of the premises or the conclusion will be omitted. But this is usually so because whatever is omitted is so obvious that it does not need to be stated. Nor does the proposition that every syllogism contains only three terms contradict the previous statement that every syllogism has three propositions, each having two terms. Each individual proposition has its own subject and predicate terms, but for the argument as a whole each term will occur twice.

Referring back to the middle term, it will be observed that in any given syllogism this may occur in the two premises in any given argument in one of four possible ways. The position of the middle term gives what is known as the *figure* of the syllogism. The four figures are as follows:

(1)	(2)	(3)	(4)
M<P	P<M	M<P	P<M
S<M	S<M	M<S	M<S
∴S<P	∴S<P	∴S<P	∴S<P

As concrete examples of these we have the following:

(1) All acids make blue litmus paper turn red;
This substance is an acid;
Therefore, this substance will make blue litmus paper turn red.
(2) All printers are members of the union;
Smith is not a member of the union;
Therefore, Smith is not a printer.
(3) All music is pleasant;
All music is sound;
Therefore, some sounds are pleasant.
(4) All physicians are mortal;
All mortals are subject to sickness;
Therefore, some who are subject to sickness are physicians.

Each of the above syllogisms is valid. But in syllogistic reasoning it must be remembered that when we ask whether a given conclusion is *valid* or *invalid* we do *not* mean, *Do you accept this conclusion as true* (i.e., as being in accord with the facts) ?, but we mean, *Does this conclusion follow from the assigned premises?* Conclusions may be valid, in the above sense, while being false in the ordinary (above) sense. The following is an illustration of an argument which is valid, but untrue:

All reasoning is a process of inference which gives us novel conclusions;
Syllogistic inference does not give us novel conclusions;
Therefore, syllogistic inference is not reasoning.

Most conclusions can be arrived at in two different ways: either by deducing the conclusion from some earlier premises, or by induction from particular cases. When we employ the deductive method we usually do so because we believe that the person to whom we present the argument is willing to grant our premises, and we want to convince him that the conclusion (the certainty of which we may accept on inductive as well as deductive

grounds) is really implied in the man's body of accepted beliefs. These comments serve to emphasize our previous point that in actual, complete thinking induction and deduction dovetail into each other.

Just as it is possible for a conclusion to be (*a*) *formally valid* while *materially false,* so it is also possible for a conclusion to be (*b*) *materially true* but *formally invalid.* In case (*a*) the premises, as inductions, are false while the deductive reasoning is correct; in case (*b*) the deductive reasoning is incorrect. The problem of establishing the truth of the premises is the problem of inductive logic, which we will take up in Part IV. At present we are concerned with the organization of the knowledge we already possess, and our problem now is, How shall we get out of our present knowledge whatever implications are contained therein? To get out consistent implications we must be familiar with the rules of correct procedure in deduction. We will now state these rules:

(1) *Every categorical syllogism contains three and only three propositions.*

(2) *In every valid syllogism there are three and only three terms.* Once they are defined, these terms must be used in the same sense throughout the syllogism. This is simply the law of identity. A violation of this law gives rise to the *fallacy of four terms,* or *ambiguous middle,* as for example:

> Transitory things should not be taken seriously;
> Life is transitory;
> Therefore, life should not be taken seriously.

(3) *The middle term must be distributed at least once in the argument.* It is not necessary that the middle term be distributed in both premises in which it occurs, though it sometimes is. As an illustration of this fallacy we have:

Every honest man attends to his business;
This person attends to his business;
Therefore, this person is an honest man.

(4) *No term should be distributed in the conclusion which was not distributed in the premise in which it occurred.* Since there are two terms in the conclusion, violations of this rule give rise to two fallacies: the *fallacy of illicit major* and the *fallacy of illicit minor.* As an example of the fallacy of illicit major we have:

All Chinamen are Mongolians;
No Japanese are Chinamen;
Therefore, no Japanese are Mongolians.

As an example of the fallacy of illicit minor we have:

All philosophers are debtors to Germany;
All philosophers are lovers of peace;
Therefore, all lovers of peace are debtors to Germany.

(5) *From two negative premises no conclusion can be drawn.* If you argue as follows:

No minor has a vote;
John is not a minor;
Therefore, John has a vote,

you are arguing invalidly. The conclusion may be true, but it is not necessarily true. We have here the *fallacy of two negative premises.* We must remember, however, that a proposition may *appear* to be negative, but in fact be affirmative.

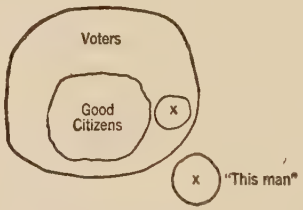
(6) *If one premise be negative, the conclusion must be negative;* and to prove a negative conclusion one of the premises must be negative. On the other hand, if both premises are affirmative, the conclusion must be affirmative. The fallacies which may arise in connection with this rule are: the fallacy of an affirmative conclusion

from a negative premise, or a negative conclusion from affirmative premises.

If one is careful to follow these rules he will not deduce invalid conclusions. It is an interesting question to discuss *why* these rules are true, or why, if one follows the rules, the conclusions will be valid. One way to state the matter is to say that a conclusion is valid if it does not violate the rules. In other words, an argument is correct if there is nothing wrong with it. But why is this the case? The text-books usually give reasons why violations of these rules are fallacious. But these answers are almost always specious explanations. What is it ultimately, we want to know, which makes the rules necessary? To answer that question would call for a philosophical discussion. This we must postpone until the next chapter. We will now only suggest how the use of Euler's circles may illustrate visually the underlying logical principles. Let us employ the following example:

All good citizens are voters;
This man is not a good citizen;
Therefore, this man is not a voter.

By Euler's circles we have:



In this example we know that the conclusion does not follow, for our circles may satisfy the premises, and yet, *as experience has shown,* "this man" is just as likely to fall inside, as outside, the class of voters. If you can draw two circles, each representing one of the premises,

while yet drawing the circle representing the conclusion at some place other than that specified by the conclusion-proposition, the argument is invalid. Thus all arguments can be tested by the rules or by the use of circles. According to this view, in the last analysis all the rules refer back to experience—to the fact that in the factual world where these propositions find their concrete embodiments we can only get valid results when the experiences which correspond to the premises are accompanied by the experiences which are stated by the conclusion.

But this interpretation of the validity of deductive inference is not the only possible explanation of "logical necessity." We bring up this matter at the present time in order to prepare the way for the more generalized discussion of consistency in the following chapter. We will only point out now that those who object to grounding logical necessity on the appeal to experience argue that this removes all real necessity from the realm of logic.

We have already explained what is meant by the *figures* of the syllogism. Logicians also speak of the *mood* of the syllogism, and we must consider this next.

The mood of a syllogism refers to the quality and the quantity of the propositions employed. Since there are four possible propositions, the A, E, I and O, the number of possible kinds of moods will be determined by the combination of each of these in the three propositions of the syllogism. We have sixty-four possible kinds of moods. The following is the traditional illustration of a syllogism of the first figure in the AAA mood.

All men are mortal;
Socrates is a man;
Therefore, Socrates is mortal.

Many of these possible moods may be ruled out at once as invalid in all figures because they violate one or more

of the previously given rules of the syllogism. Some moods will be valid in certain figures and invalid in others. In the case of arguments which are valid in one figure, it is possible to "reduce" that figure to the others. All of which indicates how rich in possibilities for mental exercise the syllogism is, and why the Medievals found deductive reasoning such a fascinating game. But in modern times we have other problems to occupy our time, so that the full details of the syllogistic technique will not be touched upon. The student now knows enough to test the validity of any actual argument in the form of a categorical syllogism, and that is sufficient for the purposes of ordinary logic.

There remains only to be discussed the more complex forms of deductive reasoning known as the *sorites,* the *hypothetical,* and the *disjunctive* syllogisms. To these we now turn.

The *Sorites.*—Sometimes two or more syllogisms will be united by having a proposition common to both of them. This type of argument does not violate our rule that every syllogism must contain only three propositions and terms, for the chain can be broken up into distinct syllogisms which follow the ordinary rules of the categorical syllogism. Here is an illustration:

All timid men are suspicious;
All superstitious men are timid;
Therefore, all superstitious men are suspicious.
But some college graduates are superstitious;
Therefore, some college graduates are suspicious.

In this case the conclusion of the first syllogism (the *pro-syllogism*) is the major premise of the second syllogism (the *epi-syllogism*). In other cases the conclusion might establish the minor premise of the other syllogism.

If we have a chain of such syllogisms and do not put the conclusion in words we have a *sorites.* Here we omit

everything except the premises until we draw the final conclusion.[1] The following is a symbolic and concrete example:

A<B	No strike but injures trade,
B<C	And consequently impoverishes the country;
C<D	But this is to diminish the means to happiness;
D<E	And as everything that diminishes the means to happiness is to be condemned,
∴A<E	Strikes must therefore be condemned.

If one accepts all the premises of this argument, the conclusion will be accepted as valid and true. The argument can be broken up into its distinct syllogisms and tested according to the rules previously given. In symbolic form we have:

First Syllogism	Second Syllogism	Third Syllogism
All B<C	All C<D	All D<E
All A<B	All A<C	All A<D
∴All A<C	∴All A<D	∴All A<E

There are two forms of the sorites. In the illustration we have just employed we have an example of what is known as the *Progressive* or *Aristotelian* sorites. Here the subject of the first premise is the subject of the conclusion and the predicate of the conclusion is the predicate of the last premise. In this case we pass from the least inclusive class to the most inclusive class. The second form of the sorites is known as the *Regressive* or *Goclenian* sorites (after Rudolf Goclenius, its formulator). In this form the subject of the last premise and the predicate of the first are linked together in the conclusion. The following is an example:

[1] The sorites is also known as a *compound enthymeme.* An enthymeme is a syllogism with one of the propositions missing. The unstated proposition may be one of the premises or the conclusion. It is left out because it is so obvious from the context that it does not require explicit statement.

All A<B	All men are vertebrates;
All C<A	All Americans are men;
All D<C	All inhabitants of the U. S. are Americans;
All E<D	All New Yorkers are inhabitants of the U. S.;
∴All E<B	Therefore, all New Yorkers are vertebrates.

It will be noted that all sorites are in the first figure. In both forms of the sorites not more than one premise can be negative or particular. In the Aristotelian sorites only the first premise may be particular and only the last may be negative. In the Goclenian sorites only the first premise may be negative and only the last may be particular.

Hypothetical Reasoning—All categorical syllogisms can be put in the form of the hypothetical syllogism. In mathematics and pure logic this is advisable, but in dealing with the factual world the generalizations which we use in our deductive arguments are regarded as sufficiently well established to render translation into hypothetical form unnecessary. A hypothetical proposition contains a categorical proposition prefixed by a statement of the condition on which the truth of the categorical depends. The conditional member of the argument is called the *antecedent* and the categorical part is called the *consequent*.

The hypothetical syllogism may take four forms, as follows:

(1) If A<B, C<D;—If a socialist is elected a revolution will occur;
A<B —A socialist was elected;
∴C<D —Therefore, a revolution did occur.

(2) If A<B, C<D;—If a socialist .....;
A⊄B —A socialist was not elected;
∴C⊄D —A revolution did not occur.

(3) If A<B, C<D;—If a socialist .....;
C<D —A revolution did occur;
∴A<B —A socialist was elected.

(4) If A<B, C<D;—If a socialist .....;
C$\nless$D —A revolution did not occur;
$\therefore$A$\nless$B —A socialist was not elected.

In the above examples the *first* and the *last forms are valid,* while the *second* and *third forms are invalid.* Generalizing from these, we derive the rules: (a) *affirming the antecedent or denying the consequent are valid forms;* (b) *denying the antecedent or affirming the consequent are invalid forms of the hypothetical syllogism.*

The reason for the rules is readily understood. The type of correlation which exists between the antecedent and the consequent is what is known as a many-one correlation. In a hypothetical argument the assertion of the antecedent is a sufficient ground for the assertion of the consequent, but the assertion of the consequent is not a sufficient ground for the assertion of the antecedent, *for the consequent may be asserted on other grounds than that given in the antecedent.* The same type of correlation makes it invalid to deny the antecedent and expect to get some conclusion.

This point can be made in another way by converting the hypothetical syllogism into a categorical syllogism. Consider the following hypothetical syllogism:

> If a student studies logic he will become a clear thinker;
> John Smith is studying logic;
> Therefore, John Smith will become a clear thinker.

In this form the argument is valid, because we are affirming the antecedent. Now perform one of the prohibited operations, such as affirming the consequent, and the conclusion becomes invalid. If we convert this into a categorical syllogism the invalidity of the conclusion will appear in the form of a violation of one of the rules of the categorical syllogism (in this case the fallacy of un-

distributed middle). Here is the corresponding invalid categorical syllogism:

> All students of logic become clear thinkers;
> John Smith is a clear thinker;
> Therefore, John Smith is a student of logic.

We must also point out that there are hypothetical arguments which differ somewhat from the foregoing patterns in that the premises and the conclusion are all hypothetical propositions. Here is an example:

If a strike is declared, poverty will increase;
If poverty increases, the people of the city will become discontented;
If the people become discontented, the political party in office will be defeated in the next election;
Therefore, if a strike is declared, the party in office will be defeated in the next election.

Such hypothetical sorites, as they are called, can be put in the form of the categorical sorites. The validity of such arguments can be tested by the rules of either the categorical or the hypothetical syllogism.

Disjunctive Reasoning.—A disjunctive syllogism is one which has a disjunctive proposition as its major premise. A disjunctive proposition states an alternative: A is either B or C. The following forms are possible:

(1) A is either B or C— He must either sink or swim;
A is B — He will sink;
∴A is not C —∴He will not swim.

(2) A is either B or C— He must either sink or swim;
A is C — He will swim;
∴A is not B —∴He will not sink.

(3) A is either B or C— He will either sink or swim;
A is not B — He will not sink;
∴A is C —∴He will swim.

(4) A is either B or C— He will either sink or swim;
A is not C — He will not swim;
∴A is B —∴He will sink.

All these forms are valid. Of course, more complicated forms are possible. A may be either B, C, D. . . . In such cases the affirmation that A is B (or C, or D . . .) means the denial of the possibility that it is any of the remainder; but the denial of one (A is not B, . . .) gives as a conclusion another disjunctive proposition containing the other alternatives.

The main difficulty with this type of argument consists in establishing the major premise. If the alternatives are not exhaustive, and true alternatives, the conclusion does not follow. Referring to our illustration, we see that we must be sure that the man must either sink or swim. But perhaps there is in fact a third possibility—such as floating. This gives rise to a nice problem of definition: is floating a form of swimming?

The *Dilemma.*—According to popular usage, to be in a dilemma is to be faced with alternatives each of which is unwelcome. In technical terminology the dilemma is defined as a type of reasoning which results from the combination of hypothetical and disjunctive propositions.

There are four possible types of the dilemma. The first form, called the *simple constructive dilemma,* yields a categorical conclusion, as follows:

> If A is B, C is D; and if E is F, C is D;
> But either A is B, or E is F;
> Therefore, C is D.

As a concrete example we have:

> If a man minds his own business he will be criticized;
> And if he minds other people's business he will be criticized;
> But either he must mind his own business or he will mind other people's business;
> Therefore, no matter what he does, he will be criticized.

In these cases the consequences of the hypothetical propositions in the major premise are the same.

The second form is the *complex constructive dilemma.* In this type the consequents of the major premise are not the same. It will also be noted that, unlike the simple dilemma, the conclusion of this argument is disjunctive. Here is an example:

If a statesman who sees his former opinions to be wrong does not alter his course he is guilty of deceit; and if he does alter his course he is open to the charge of inconsistency;
But either he does not alter his course or he does;
Therefore he is either guilty of deceit, or he is open to the charge of inconsistency.

The third form is the *simple destructive dilemma.* This form gives a categorical conclusion. Since the major premise has one antecedent and two consequents, and these consequents are denied in the minor premise, the antecedent is therefore denied in the conclusion. A concrete example of this is found in the famous argument of Zeno to demonstrate that motion is impossible:

If a thing moves, it must either move in the place where it is or in the place where it is not;
But it cannot move where it is, nor can it move where it is not;
Therefore, it cannot move.

The fourth form of the dilemma is the *complex destructive dilemma.* Here the conclusion is composed of two negative propositions disjunctively related. The following is a classical illustration of this type:

If this man were wise he would not speak irreverently of the Scripture in jest; and if he were good he would not do so in earnest;
But he does it either in jest or in earnest;
Therefore, he is either not wise or not good.

While the dilemma is seldom a logically convincing type of argument, when it is employed by a clever speaker or writer it may be made embarrassing to an opponent. In constructing a dilemma the difficulty comes in estab-

lishing the major premise. It is not often that the alternatives presented exhaust all the possibilities. Moreover, there is the possibility that the alternatives offered are not necessarily exclusive of each other. In other words, the dilemma meets all the difficulties of both the hypothetical and disjunctive modes of reasoning.

To refute a dilemma the following avenues of attack are open: (*a*) One may *escape between the horns of a dilemma,* which would otherwise "gore" the person. To do this one must show that there is some other alternative which was overlooked. (*b*) The dilemma may be *taken by the horns,* accepting the alternatives proposed, and then showing that one or both the consequences which are asserted do not follow. If the original dilemma can be attacked on these grounds it is possible to formulate a *counter-dilemma* containing a conclusion which is contradictory to the original conclusion. Thus this instrument of dialectical strategy is a dangerous form of argument which may be turned against the creator.

A Critique of the Syllogism

Now that we have discussed the forms and uses of syllogistic inference, we must consider some of the criticisms which have been made of this type of reasoning.

One of the commonest criticisms of syllogistic reasoning is that it teaches us nothing new. As an illustration, consider what we do in the following syllogism, which epitomizes Schopenhauer's pessimistic philosophy of life:

> All striving is suffering;
> All living is striving;
> Therefore, all living is suffering.

Examination of the conclusion shows us that it is implicitly contained in the premises, and that in syllogistic inference we merely draw out that which is already "there."

Since, the critics add, reasoning is creative, bringing us to novel conclusions, we cannot regard deductive inference as real reasoning. In reply the defenders of deductive reasoning will urge that while it is true that the conclusion is *logically* contained in the premises, man is not entirely a logical animal. He is a psychological creature as well, and needs to make explicit (by going through the various steps of the syllogism) that which is implicit. The fact that persons sometimes make mistakes proves that psychologically we arrive at something which was not immediately manifested to us when the premises were first stated.

This is one criticism which is brought against the syllogism. A somewhat different indictment has been brought against this type of logic by F. C. S. Schiller, the English pragmatist. On various occasions [2] Schiller has reiterated his contempt for *formal* logic. As part of the explanation for this disparagement we find him approving the criticism that it is based upon an infinite regress. Syllogistic proof presupposes the truth of its premises. If these are questioned, they must be proved. But to prove them requires the proof of a larger number of premises, which in turn need to be proved—and so on. Thus the more you try to prove the more you have to prove and the further you get from proof.

In earlier days mathematics escaped this criticism by claiming to be based upon *axioms* which were necessarily true, and to which there were not admitted to be any alternatives. As Plato, Descartes, and others said, these axioms are *intuitive*. But no one believes this now, says Schiller. At best, *axioms are only postulates,* and you don't have to accept them if you prefer not to. And according to the Schillerian viewpoint, the same thing has

[2] In his book, *Formal Logic,* and in an article on "The Infinite Regress of Proof," *Mind,* Vol. 37 (1928), pp. 353-354.

happened in logic. This leads, it is said, to a pragmatic conception, where absolute truth is rejected.

Here again the defenders of syllogistic logic have their reply ready. They will point out that the fact that we may not get absolute certainty through this type of reasoning is no grounds for rejecting it. We may get probability in our conclusions through the probable truth of our inductions, which constitute the premises of syllogistic arguments.

We see, therefore, that none of the foregoing criticisms is completely disastrous. It still remains true that the syllogism is a valuable and valid device for showing the implications between previously established generalizations. However, there is another type of criticism of the traditional formulation of deductive reasoning which we must consider. This is the criticism which points out that in the past deductive logic has been restricted to a special type of inference, the syllogism, whereas the truth of the matter is that deductive logic is much broader than the syllogistic patterns would lead us to believe. Accordingly, a truly adequate treatment would be much more general.[3] We will here merely indicate the direction which this generalization would take.

We have already referred to the fact that there are various types of relations. Thus far we have employed the relation of class inclusion more than any other. The various possible kinds of relations can be grouped into several classes, of which the following are the two main types:

(A) Symmetrical and Asymmetrical
(B) Transitive and Intransitive

[3] In his book, *The Ways of Knowing,* Professor W. P. Montague argues (pp. 93-99) that the Aristotelian syllogism can be reduced to one species of a generalized genus of relational syllogisms. This general logic of relatives he terms the "hyper-syllogism."

Symmetrical Relations.—A relation, "R," is said to be symmetrical when it reads both ways: a symmetrical relation is one which is equal to its converse. Symbolically this is represented thus: $xRy < yRx$. The relations of equality, difference, consistency, contradiction, and others, are symmetrical.

Asymmetrical Relations.—An asymmetrical relation is one which reads only one way; that is, when "R" is such that yRx is false when xRy is true. Such relations as, precedes, greater than, etc., are asymmetrical.

Transitive Relations.—If, as soon as a relation, "R," arises between x and y, and y and z, it also arises between x and z, the relation is said to be transitive; the relation, in other words, carries across the middle term. Symbolically this is represented thus: $(xRy)\ (yRz) < (xRz)$. Temporal relations, relations of magnitude, spatial relations (ordinarily), etc., are transitive. If A precedes B, and B precedes C, then A precedes C. The "axiom" of mathematics that things equal to the same thing are equal to each other is based upon the transitivity of the relation of equality.

Intransitive Relations.—These are relations which do not carry over. Such relations as, is father of, loves, etc., are intransitive. If we employ *Boole's Law,* which states that the number of possible classes which result from the process of division is equal to 2^n (where n is the number of factors involved), it is at once obvious that we have four possible kinds of terms with respect to the foregoing classes. If relations are:

Symmetrical or Asymmetrical—(R = T or T′)
and
Transitive or Intransitive —(R = S or S′)
then, $R = TS + TS' + T'S + T'S'$
↓ ↓ ↓ ↓
"equals" "taller" "unlike" "hates"
"implies" than"

It will now be obvious that the syllogism, which employs the relation of "implication" or "class inclusion," is an illustration of a transitive relation. That is $(x<y)(y<z)<(x<z)$. But the reduction of the syllogism, as one special device for exhibiting the implications of propositions, to the generalized notions of symbolic logic cannot be undertaken here.[4]

[4] The foregoing analysis of relations takes care of arguments ordinarily termed "extra-syllogistic." Such arguments can be put into syllogistic form when anything is to be gained by it.

Chapter VI

THE FALLACIES OF DEDUCTIVE LOGIC

IN connection with deductive logic it is customary to enumerate and classify the various types of fallacies which can occur. Classifications in this field do not always agree, but differences of treatment are usually confined to minor points. The following table gives us a list of the various fallacies.

First of all let us consider the fallacies which are ascribed to "errors of interpretation." We will here say nothing further about illogical conversion and obversion, since they have already been discussed.

The *fallacy of amphiboly* (or *amphibology*) occurs when, because of an ambiguity, a sentence may be interpreted in several ways. This usually results accidentally (i.e., is not intended), but a sentence guilty of this may be adopted consciously by someone who finds it to his interest to do so. It will be remembered that the decrees of the oracle at Delphi were of this character. We find a concrete example of amphiboly in Shakespeare's *Henry VI,* in the prophecy that "the Duke yet lives that Henry shall destroy."

The *fallacy of accent* arises as a result of misleading emphasis. It is interesting to note how many different interpretations may result from emphasis upon different words in the same sentence. Writers and speakers who lift sentences from their contexts for purposes of quotation run the risk of committing this fallacy.

The *fallacy of composition* occurs when one invalidly passes from a proposition which is true for the individu-

als of a group (or which is true in a distributive sense) to the assertion of the truth of the proposition as applied to the group as a whole. As an illustration of this we have: Each member of the all-American team is the best man for his position; therefore the team as a whole is the best in the country.

FALLACIES

Errors of Interpretation

1. Illogical Conversion or Obversion
2. Amphiboly
3. Accent
4. Composition
5. Division

Mistakes in Reasoning

Formal	*Material*
A—In Categorical Arguments	1. Begging the question
1. Four terms (including ambiguous middle terms)	2. Reasoning in a circle
2. Undistributed middle	3. Complex question
3. Illicit major	4. Argumentum ad hominem
4. Illicit minor	5. Non sequitur and irrelevant conclusion
B—In Hypothetical Syllogisms	
5. Denying the antecedent	
6. Affirming the consequent	
C—In Disjunctive Arguments	
7. Imperfect disjunction	
8. Dilemmatic fallacy	

The *fallacy of division* is the converse of the fallacy of composition. The fallacy consists in supposing that what is true of the whole most therefore be true of the parts. If, from the truth of the proposition that the army as a whole is weak, we infer that therefore some particular company is weak we commit this fallacy.

Coming now to the *formal* and *material* fallacies we may point out that this distinction rests upon the following interpretation: a *formal fallacy* is an error in the structure of the argument; even if both premises be true the conclusion is invalid; or in other words, a formal fallacy is a violation of the formal rules of the syllogism. A *material fallacy* is said to be one which arises in the subject matter of the argument. Material fallacies are sometimes called *fallacies of assumption.*

We have already discussed the formal fallacies. But material fallacies have not been dealt with, and we must now consider them.

Begging the question is technically known as *petitio principii.* This fallacy arises when an argument to prove a position assumes the thing to be proved. Thus if a behaviorist argues that mind cannot be a reality because it cannot be measured quantitatively he is begging the question: he is assuming that in order that an entity may be real it must be measurable, which remains to be proved.

Reasoning in a circle is a special case of begging the question. If, to refute a behaviorist, a person argues that man must be a conscious being because he has mental states the advocate of the view is obviously reasoning in a circle. Such reasoning makes no progress and is to be avoided.

The *fallacy of complex question* is the interrogative form of begging the question. If a student is asked, "Have you begun to study yet?" an answer in terms of "yes" or "no" implies the admission of the point which is assumed. This may not do justice to the actual facts in the case. The "leading questions" which lawyers use in examining witnesses in court are frequently of this type. They are very properly objected to.

The *fallacy of argumentum ad hominem* results when an attempt is made to establish, or discredit, a proposi-

tion by appealing to the emotions or prejudices of the persons to whom the argument is addressed. One may attempt to gain credence for a view by casting aspersions on those who take the opposite view, by appealing to the authority of a great name or the authority of tradition, or by appeal to the sympathy or animosity of men, and in other ways. Here the value of a proposition or doctrine is no longer decided on the basis of its logical merits. The attainment of truth, however, calls for an abandonment of such irrational appeals.

The *fallacy of non sequitur and irrelevant conclusion* are similar, the latter being an extreme case of the former. In both cases the conclusion does not follow from the premises. In some cases the stratagem is deliberately employed of finding several premises which contain the terms of the conclusion and arranging the propositions in what appears to be logical sequence. Those who are not on the alert may be "taken in" by this dialectical device. The following is De Morgan's illustration of this:

Episcopacy is of Scripture origin;
The Church of England is the only Episcopal church in England;
Therefore, the church established is the church which should be supported.

Chapter VII

SCEPTICISM, CONSISTENCY, AND BELIEF

IN previous chapters the reasons have been given for holding that logical consistency is an achievement as excellent as it is rare. But the question as to the test of consistency was left unanswered. In approaching the conclusion of this discussion of the organization of our beliefs we must consider in more detail the nature and the criteria of consistency.

The question of what constitutes the essence of consistency cannot be answered as easily as might be supposed. Leaving to the side for the moment the problem of the criteria of consistency, let us note at the outset that consistency is obviously a relation between ideas, propositions, and chains of reasoning. Two propositions may be said to be consistent with each other when the truth of one implies the truth of the other. On the other hand, two propositions (or ideas) are inconsistent with each other when the truth of one implies the falsity of the other. Let it be noted, however, that this last statement does not mean that because (or if) there are true propositions there necessarily are false propositions; for it is possible (though extremely improbable) that all propositions might be true. This no doubt appears paradoxical, so, in order to clarify the statement, let us go back a bit.

Logicians who have dealt with the purely formal aspects of implication have pointed out that necessary inference is summed up in the form, *p implies q;* or putting it in hypothetical form,—*if p then q.* According to one interpretation, the statement *p implies q* means that either

p is false or *q* is true. That is to say, in order that any proposition may be false there must be true propositions. Or in other words, it is impossible for all propositions to be false, since the proposition that all propositions are false is stated to be true. We see, therefore, that the existence of falsity implies truth, but the existence of truth does not necessarily imply falsity.

This may appear to be a strange conclusion. But the situation is not without parallel. The situation in logic is apparently analogous to the relation between light and darkness (an analogy which Plato made the most of). The existence of darkness implies light (since darkness is the absence of light), but the existence of light does not imply darkness. This conclusion is important, for, as we have already observed, in it is contained the reply to the thoroughgoing scepticism which some iconoclasts of certainty have attempted. It is impossible to be a thoroughgoing and consistent sceptic, for the person who says that there is no truth is implicitly contradicting himself. The consistent sceptic would not be able to deny the existence of truth, for he would already know more than his theory permits. To escape contradicting himself the sceptic would have to say: I don't know whether there is any truth. But, as others have pointed out, even this is not consistent: the sceptic at least knows that he does not know—which is also more than he is supposed to know. What the sceptic should say is: I don't know whether I don't know whether I don't know . . . *ad infinitum*.

All of which merely proves that there must be some truth. But there is this much to be said for scepticism: It is quite likely that most individuals greatly overestimate the amount of truth which they possess. Particularly those people with the "will to believe" are prone to exaggerate the amount of truth they think they have.

Probably, as Bertrand Russell has pointed out, a more sceptical attitude would be salutary, especially in those fields where cocksure fanaticism holds sway over men's minds. Other things being equal, the man who believes less than his neighbors is less likely to be inconsistent, and less likely to be dogmatic. If his scepticism is not carried too far, such an individual would presumably make a better citizen of the state. It would be an interesting social experiment to try to run a society composed of individuals who had been inoculated with a large injection of scepticism. *Evidently there must be some golden mean in beliefs, lying somewhere between the extreme of no belief, on the one hand, and the other extreme of belief in everything, on the other hand.* But how is one to find this happy logical medium which gives the optimum of consistency with the minimum of inconsistency!

We now come to the question of why it is that our beliefs are consistent or inconsistent, compatible or incompatible. One famous answer to this goes back to the formal interpretation of the "laws of thought." In this view, consistency, as a relation between mathematicological propositions, has no necessary reference to the natural or physical world. Many mathematicians regard their science as concerned with logically possible domains. That is, the concepts, postulates, and theorems of mathematics are not held to have any necessary reference to empirical reality, the concrete applications of arithmetic, algebra, and geometry being incidental to pure mathematics. Inferences are then held to be consistent with the propositions or postulates (in earlier times called "axioms") which constitute the starting point because there is a kind of conceptual determinism which compels us to accept the derived theorems or propositions of mathematics and logic. Mathematics, according

to a famous definition, is the science of necessary inference—necessary because the laws of thought are the laws of the ways in which the mind must operate.

This view is taken by Professor C. J. Keyser,[1] who tells us: "Mathematics is, in a word, the study of Fate. Let me hasten to add that the Fate is not physical—it is spiritual—the unbreakable binding thread of destiny runs through the universum of rigorous thought: the fate is logical Fate." Essentially the same idea is suggested by Professor G. N. Lewis.[2] The similarities in the non-Euclidian geometries advanced independently and almost simultaneously by Lobachevski and Bolyai suggests, as Professor Lewis states, that "we cannot avoid the thought that having embarked upon a certain line of mathematical inquiry, while we appear to have preserved the utmost personal freedom, we seem bound to follow certain paths and to make and remake certain discoveries, just as we do in physics or chemistry. Is there, then, almost as much an objective world of mathematics as there is an objective world of physics?"

It is perhaps a tragedy that such a beautiful theory should not meet with general acceptance. But the history of human thought (not to mention insane asylums!) shows us that all minds do not work in the same way. Indeed, when one comes to think about it, what right *do* we have to demand that certain propositions should be inconsistent for all minds simply because they are inconsistent for some? For example, if the theologians assert that God is all-wise and omnipotent (as affirmed in the Westminster Confession) and then assert that God is not the author of evil, can we with absolute confidence accuse them of being inconsistent? Again, what right have we to assert that the same body cannot be in two different

[1] *Mathematical Philosophy*, p. 5.
[2] *The Anatomy of Science*, p. 41.

places at the same time simply because we accept the proposition that a thing is where it is?

If we confine ourselves to the second of the above questions the answer which suggests itself as a solution to our general problem is this: Propositions are consistent with each other when experience has repeatedly shown us that the facts affirmed by the several propositions actually go together in verifiable situations in the external world. Mathematicians subscribe to this common-sense version of consistency when, in order to test a set of postulates, they say: Find an *interpretation;* that is, apply the postulates to some actual field, and if the postulates are in accord with what experience there reveals the postulates are valid and consistent. As an example we may take the following argument in its formal aspect: If A bears a certain relation to B, and B bears that same relation to C, then A bears that relation to C. Whether this inference is valid or not depends on what the specified relation is. Experience tells us that if A *is north of* B, and B is north of C, then A is north of C (excluding going over the "top" of the world). Here the transitive character of space relations, *discovered by experience,* makes the conclusion consistent with the premises. But if the relation is *hates,* the conclusion that A hates C, because A hates B and B hates C, is not in accord with all observations and therefore is inconsistent with the original propositions. Experience (or a concrete interpretation) is the court of last appeal, says the empiricist.

But all logicians would not accept this. Those who ground consistency on the inviolability of the "laws of thought" will point out that to say an "interpretation" gives us the test of the consistency of beliefs or propositions is to suppose that the interpretation itself is consistent, that the world revealed by experience is logically coherent. Thus we seem to be going around in circles. We seem to have arrived at the point where we cannot

know what specific propositions are consistent with each other because we do not know what the test of consistency may be.[3] This might please those persons who would like to believe emotionally satisfying, though logically inconsistent, propositions,—such as, perhaps, a literal interpretation of the Bible, including the belief that the earth is flat and that Joshua stopped the sun, while still holding to the established facts of science, such as that the earth is round and revolves on its axis.

Let us hope that the situation is not so chaotic as that. It seems to me that the answer is quite simple. Both the above views are correct, each within its own sphere. In mathematics and logic consistency is a matter of definition. You are free to define anything (except, perhaps, the laws of thought) in any way you please, but having defined these entities as you please you must not attribute contradictory qualities to them. Since time does not enter into the ideal domain of mathematics, a thing is what it is defined to be so long as you want it to remain what it is. But in natural science, as we have seen, definitions must fit the facts. The consistency of propositions about this stubborn physical universe is established by a determination of the correspondence of these propositions with these facts. It is possible, of course, that nature is fundamentally irrational; that, at least, she contains some measure of inconsistency. If this is so this means a definite limitation upon the existence or scope of science. It means also that the processes of nature are not reducible to a simple set of postulates

[3] Professor J. W. Young, in his *Lectures on the Fundamental Concepts of Algebra and Geometry*, tells us (p. 44): "At the present time, however, no absolute test for consistency is known." Later (p. 224) he says: "We have just recalled the fact that the important question of the logical consistency of a set of assumptions can be determined only by exhibiting a concrete application. This is a fatal defect in any attempt to place the foundations of mathematics on a purely formal basis . . . We have here a problem of great difficulty which can certainly not be solved by means of formal logic alone. It is of a metaphysical nature; its solution must be sought in the field of philosophy."

which are complete, consistent, and categorical. We would certainly like to believe that nature is consistent and understandable, but for natural science the *"law"* of *sufficient reason* and the *principle of the uniformity of nature* must always remain postulates. We will have more to say about this in connection with induction.

Perhaps it is unfortunate that we must be left with two worlds to deal with; dualisms are always disconcerting. But one may entertain the faith, if he wishes, that some day these two domains will be amalgamated. Einstein is in fact trying to accomplish this very feat in what has been called the "physicizing of geometry" or the "geometrization of physics." Perhaps when the last word has been said, the theoretical principles of consistency will be found to be identical with the laws of ultimate physical reality. No one has yet succeeded in reducing the laws of physics to the laws of epistemology, but it is at least a possible point of view that the interpretations of logic and mathematics apply to nature because nature is ultimately rational and intelligible, and because man's processes of reasoning are illustrations of physical laws operating in his own thinking brain. But with faiths we now have no further concern.

Analysis of the opinions expressed by persons who like to pose as authorities on any and all subjects will reveal the fact that many inconsistent beliefs are entertained by these "know-it-alls" simply because social pressure is not brought to bear to force such persons to reason out their views systematically. This observation suggests the following question: Is it not possible to devise a consistency test which, like intelligence and trade tests, will make it possible to measure the logical consistency of individuals and, by comparison of the performance with previously established norms, estimate the consistency of the individuals tested?

Suppose state legislators advocating anti-evolution laws were brought face to face with the following proposition: *Only the expert in any field*—e.g., in electrical engineering—*is competent to make judgments about the truth of the propositions within that field.* It requires no great optimism to suppose that most legislators would subscribe to the truth of that proposition. Now let us suppose that at some later date our legislators are confronted with this proposition: *State legislators are competent to pass judgment on the question of whether the doctrine of biological evolution is a fact.* If the answer coincides with the practice of the anti-evolution legislators, these legislators would have to admit their acceptance of the last proposition. But when the two propositions are set side by side, is it not at once evident that the two are inconsistent? One or the other must go, for the problem of the truth of the doctrine of biological evolution is a matter which only the experts can decide. Intellectual honesty therefore calls for the abandonment of one or the other of these two propositions.

This illustration may make it evident that one of the reasons why we are all, to a greater or less degree, inconsistent is that we do not set down our articles of faith in black and white, so that our inconsistencies, like our sins, will find us out.

As a result of the conviction that it is possible to develop a consistency test, the following sets of propositions were formulated and grouped. In this test we are trying to measure what might be called the "consistency quotient" of the individual. The general theory underlying the test is this: *If a person believes a certain proposition in one field he ought to believe* (or *disbelieve*) *another proposition in some other field, if the two propositions are logically related.* If a person believes two propositions which are clearly inconsistent with each other this

fact can then be pointed out. Proceeding on this theory three fields of human knowledge have been singled out, and within each field six propositions are given which were meant to be logically inter-dependent.

However, instead of replying to these propositions with *yes* or *no,* allowance is made for uncertainty. This is secured by assigning to each proposition a "truth-value," expressed in terms of percentage.[4] This truth-value is the degree of certainty which this proposition possesses for the individual who is rating the proposition. Thus, if one does not believe in miracles the truth-value of the proposition will be estimated as zero, 0%. If one believes absolutely that the sun will rise tomorrow the truth-value assigned to that proposition will be 100%. Degrees of probability are represented by figures between 99% and 51%, even doubt by a truth-value of 50%, and improbability by values from 49% to 1%. Thus, to be consistent, if a person believes that the evidence indicates that biological evolution is highly probable, say 95%, he ought not to give the doctrine of special creation a value of over 5%.

The propositions to be considered are given in the following table. It is left to the reader to assign to each proposition whatever truth-value he thinks it deserves.

A Test of Consistency

100%	represents	Certainty
51% to 99%	"	Probability
50%	"	Uncertainty
1% to 49%	"	Improbability
0	"	Impossibility

[4] This idea was suggested to the author by Dr. Marvin Farber, who has originated the notion of the *probability function* as a parallel concept to the notion of the *propositional function.* In his book on the *Technique of Controversy* Dr. Boris B. Bogoslovsky has also graded propositions numerically according to their probable truth.

I may add that the test here presented was first published in *Psyche* (April 1927) and in the *Forum* (May 1927).

RELIGION:

	%
God exists	
Freedom of choice is real	
Man has a soul which is immortal	
Divinity of Christ	
Fundamentalism in religion is desirable	
The miracles of the Bible are true	

SCIENCE:

	%
Science will eventually displace religion	
All men are created equal in native ability	
Man is only a complicated machine	
Uniformity of natural law	
Biological evolution	
Rotundity of earth	

POLITICS:

	%
Democracy is a failure	
Eugenics is the hope of civilization	
Prohibition is a desirable measure	
Men cannot be made moral by legislation	
The white race is superior to other races	
Criminal punishment is just	

This scheme seems to possess at least two merits: (*a*) If the questions are conscientiously answered the results will be a fairly reliable index of what the subject actually thinks on the matters considered. (*b*) It may help an individual settle his ideas on specific questions about which he has hitherto been uncertain. For example, if as a scientist one holds certain beliefs about human nature, this may help to determine what one ought to believe about propositions in the field of social ethics. Thus a determinist in psychology, who believes that human beings do not have "freedom of choice" and that heredity and environment are the two overmastering fates, ought not to hold criminals responsible for their acts, and therefore ought not to regard punishment as *just*. But it must be noted that one might regard punishment as *desirable* for other reasons—because punishment sets an example

which acts as a deterrent to others, or because it modifies the conduct of the person who is punished. This difference can be indicated in the terminology of the proposition.

Criticisms of the method of grading propositions can easily be made. It may be pointed out, for example, that one could go through the propositions giving each one a truth-value of 50%, and then claim that no inconsistencies could appear. But such a person is either a complete sceptic, or lazy, or perverse. It is interesting to note that records so far indicate that thoroughgoing sceptics are very rare. It must also be pointed out that two persons may disagree on a number of propositions and yet both be internally consistent. Thus a materialist and an idealist would grade some propositions in an inverse proportion, but until we have the final philosophy we could not decide which of these two views is correct. The present test measures *consistency* and not *truth*. Nevertheless, if the issues are clearly drawn and all the implications brought out, such a test may aid us in deciding truth as well as consistency.

This test has been given to several hundred university students. Since the test is to be revised, no detailed analysis and statement of the results will here be attempted. At this time we will only point out that the results indicate that human beings have a marked tendency to believe propositions which the experts would agree are inconsistent with each other, especially if the belief is summed up in some popular slogan or catch-penny phrase. We are all too frequently the dupes of phrase-mongers. The results seem to indicate that most people do not follow up the implications of words and propositions, but prefer the sentimentalism of orators.

It may be objected that these questions are too difficult for college students to wrestle with. The reply to this

is that if the function of education is indeed to develop in students an interest in the important problems of contemporary society and help them formulate tentative answers to these problems, then education is failing in its mission. This, let it be noted, is not the fault of the students. It indicates the viciousness of the present system of discussing each subject (economics, biology, psychology, ethics, etc.) of the curriculum as though it were a separate interest, thus leaving to inadequate philosophy departments the difficult problem of the coördination of knowledge.

If it is true, as results seem to indicate, that only about 10% of our students really think sufficiently to achieve some measure of consistency, teachers have no cause to congratulate themselves. But we may still entertain the hope that when education functions more effectively in promoting the art of thinking the results will surprise us. Fortunately, the future stretches out a long way ahead of us.

Part IV

HOW NEW KNOWLEDGE IS OBTAINED

Chapter I

THE LOGIC OF SCIENTIFIC METHOD

IT does not require any prolonged research to reveal the fact that contemporary civilization is to a large extent a product of modern science. One needs only to look about, note the presence of bridges, telephone lines, street cars, autos, and listen to the voice of the ever present radio, to confirm this statement. The entire world of material things which surrounds us and ministers to our wants is the result of the practical application of theoretical discoveries in science.

To many philosophical students of history it appears that the inventor of the steam engine and the discoverer of the laws of electromagnetism have done more to revolutionize the old order of things and create the new world than all the exhortations of loquacious theorists. These individuals will point out—what is so obvious that it needs no emphasis—that the same industrial revolution which is responsible for our present factory system of production, as well as the means of transportation whereby these products are distributed, has also been responsible for the growth of large cities, with all the perplexing moral and social problems which have accompanied their development.

The effects of science are many and varied. In our enthusiasm for the brilliant success of applied science we must not overlook the concomitant increase in intellectual power to understand and explain the phenomena of nature. Nor must we fail to observe that, diverse as are these contributions of modern science to civilization, there

is a fundamental unity underlying them all which is of significance. This common element in all science, physical as well as social, pure as well as applied, is to be found in the *technique of procedure* which is the common denominator of all science. This technique is commonly known as *scientific method.* If we are right in ascribing the success of science in controlling, predicting and explaining natural processes to the use of this method, it at once becomes obvious that an understanding of scientific method is of importance to anyone who wishes to undertake work in any science, or even understand the full significance of science in the modern world. In this present chapter it is our purpose to examine the characteristics of this method; in the following chapters we will then note in detail how this method is effectively applied by science.

It is common knowledge that in reasoning one may reach consistent conclusions from premises which are themselves false; it is also possible to come to conclusions which are inconsistent with the premises though they are in accordance with the facts. In the first case we have a fallacy of inductive logic; in the second case we have a fallacy of deductive logic. Neither type of reasoning embodies the full ideal of thought. It is obvious that *good* reasoning aims to be *materially true* (in accordance with the facts) as well as *formally valid* (consistent with the premises). The rules of procedure which enable us to achieve the second desideratum are discussed under the subject of deductive logic. The rules of procedure which enable us to arrive at the generalizations which are the basis (the premises) of our deductive arguments, thus making it possible to achieve the first desideratum, constitute the materials of inductive logic. This is the subject which we now have before us for consideration.

It is generally recognized that all thinking presupposes

certain fundamental assumptions. Since science is a form of reasoning, it follows that this will also be true for science. But in addition to these assumptions of all thought, experimental science, in investigating those ensembles of facts which form the framework of external nature, must employ other assumptions. Since part of the substance of scientific method consists in the eagerness to discover and make explicit the assumptions which are presupposed in the process of making empirical investigations, we can introduce our discussion of the subject of scientific method by an exposition of these assumptions. These may be formulated in the following manner:

THE ASSUMPTIONS OF SCIENCE

(A) 1. The Law of Identity
2. The Law of Contradiction
3. The Law of Excluded Middle

(B) 1. The Principle of Sufficient Reason
2. The Principle of Parsimony
3. The Principle of the Uniformity of Nature

(C) 1. The Principle of the Reliability of Sense Experience
2. The Principle of Correspondence between ideas (or conceptual constructs) and the objective realities of the external world
3. The Principle which states that the field of science may be enlarged to include entities and processes beyond the range of sense experience, through logical inferences from known phenomena and laws

It will be observed that these assumptions are grouped into three classes. The first three assumptions, traditionally known as the "laws" of thought, have already been discussed and nothing further need be said about them here.

The first and second principles of group B are also usually called "laws," but they are really assumptions which

it is necessary to make if inductive science is to be possible. This is explicitly recognized in the next, the principle of the uniformity of nature, which is not ordinarily designated as a "law." It would have been more satisfactory from the point of view of logic if the "law of parsimony" and the "law of sufficient reason" had also been called *principles*.

We will now give a brief statement of these principles.

The Principle of Sufficient Reason.—Everything which is, or all that happens, has a good and sufficient reason for its existence or happening. Nature, in other words, is intelligible; she will not put us to permanent intellectual confusion. Every phenomenon is capable of being understood and explained.

The Principle of Parsimony.—This principle advises us to be stingy or parsimonious in the formulation of our hypotheses. Pare away superfluous assumptions, for, other things being equal, the simplest explanation is the most likely to turn out to be true. The principle is also sometimes called Occam's razor, after William of Occam, a scholastic philosopher who said that entities (hypotheses, assumptions, etc.) are not to be multiplied beyond necessity (or *Entia non multiplicanda sunt præter necessitatem*).

The Principle of the Uniformity of Nature.—Nature is uniform in her behavior. Everything happens according to invariable laws. The same causes produce the same effects, and the same effects are due to the same causes.[1]

It is clear that these principles are on a different level from the first three laws of thought, for the laws of thought are presupposed by any thinking whatsoever,

[1] This principle, which is presupposed in the inductive establishing of causal laws, is very important, and we will discuss it at greater length in Chapter V.

while the next three postulates of science are employed only when we are investigating some set of facts open to sense experience.

The first principle of group C, that of the reliability of sense experience, tells us (*a*) that there is a world of facts independent of the mind (or the knowing organism), and (*b*) that by using his senses (and of course making due allowances for illusions and personal errors) man can find certain uniformities in sequence and interrelation among these facts. These uniformities then may be taken as the basis for the formulation of natural laws.

The statement that nature is open to investigation, that there are no inscrutable or occult mysteries, implies that nothing is too sacred to be studied and understood. This principle also implies that no phenomenon is to be taken as objectively real which does not stand the the test of repeated observation by various trained observers. This principle is important, for its acceptance marked the transition from superstition to modern science.

The next principle, that there is some kind of correspondence between our conceptual (logical) processes and the objective realities and processes of the external world, tells us that science assumes some sort of epistemological doctrine. Many scientists—the empirical grubbers, we may call them—have not been aware of this fact, but the more philosophical scientists—men like H. Poincaré, Ernst Mach, Wilhelm Ostwald, and others—were keenly conscious of the necessity for clarifying the epistemological foundations of science.

Our last principle tells us that science is restricted to that which is perceptible, or is capable of being inferred from the perceptible. This principle means (*a*) that while science is confined to a world of measurable quantities, it may (*b*) also include within its scope those non-

observed realities and processes which are verifiable derivatives of the world which is observed and measured.

Many of the objects of study of physical science have never been directly perceived. Even the ultra-microscope does not possess a sufficiently high resolving power to make the infra-universe of molecules, atoms, electrons and protons, accessible to sense perception. And yet all scientists are now agreed that these terms refer to genuine realities and not merely to metaphysical constructs, as some scientists of a day not long past affirmed.

The extent to which we may infer the existence and behavior of unobserved realities from perceived phenomena is seen in astronomy. In the study of the stars the astronomers are now speculating about the interior constitution of these stellar bodies, which of course they have never seen. Physicists also venture beyond the realm of the observed when they attempt to determine the properties of the ether of space, a reality which, if it exists, is forever beyond direct observation.

In all such cases, however, where science attempts to ascribe properties and dimensions to these unperceived objects of thought, these objects may be accepted as existent realities in so far as they are valid inferences from the world of perceived and measured facts.

As everyone knows, the scientific mood or temperament is one which does not come natural to most individuals. It is something which must be cultivated. A scientific mind is a *disciplined* mind, and this disciplining takes time, patience and effort. Since this temperament calls for certain positive and unusual qualities, it is doubtful whether all individuals are capable of being converted into great scientists. Nevertheless it is possible for anyone to improve his performance in the use of scientific method if he will but exert the necessary effort. At the very least, thought and effort devoted to the study of

this subject would bring us all to the point where we would cease to be the dupes of pseudo-scientists and charlatans. And to achieve that much is ample returns for the time and energy expended.

Let us therefore examine those qualities of mind which are essential ingredients in the individual who respects and employs this technique which is now so generally termed scientific method. There are a number of such qualities essential to the scientific temperament and the order in which they are given is somewhat arbitrary.

The Characteristics of the Scientific Mind

1. A respect for facts.
2. The critical attitude which carries with it freedom from excessive emotionality.
3. Cautiousness of statement and capacity to suspend judgment.
4. Persistence and patience in "following the argument."
5. Intolerance of obscurity and impatience with inadequate evidence.
6. A sense of the inter-relatedness of things.
7. Ability to break up a complex situation into its elements, isolate them, and avoid confusion in handling the several variables.
8. Fertility in making supplementary hypotheses after the most obvious ones have been made, tested, and abandoned.

These qualities of mind are somewhat diverse, and it is seldom that one will find them all embodied in a single individual. Some scientists are excellent at suggesting new hypotheses; they are brilliant speculators, but poor at testing. Other scientists are patient, painstaking experimentalists, who seldom have an original idea of their own, but can test the hypotheses of more creative thinkers. Men like Galileo, Newton, Darwin, Faraday, and a few others, were original and creative thinkers who were also critical and patient testers of their own ideas. Unfortunately few of us are Newtons, Faradays, or Dar-

wins. But we can at least learn to discriminate between the great and the pseudo-great, and learn to reverence those qualities in others which may be conspicuous by their absence in us.

In these days of intellectual counterfeit and quasi-scientific "isms" it is essential to be able to discriminate. For this reason the following characteristics of a "fake" article are worthy of study:

Qualities of a "Fake" Article

1. *Lack of Data.*—If the data are not valuable, if they are uncriticized, or if there is no method of checking the data, this throws discredit upon the article.
2. *Lack of Logic.*—If the author makes assumptions which are unrecognized or unjustified, or generalizes from too few facts, this indicates that the author is unreliable.
3. *Evident Advertising.*—If there is a photograph of the author, or testimonials included in the book or article, the testimonials being from persons who are not authorities in the field, this tends to discredit the article or book. If the article is published by the author and is for sale by him, this casts doubt upon the value of the contribution. Sometimes, however, reputable investigators are unable to get their works published through the usual channels and are forced to publish their ideas at their own expense.
4. *Unfamiliarity with the Literature.*—If the author is obviously unaware of the work of authorities in the field, if he fails to mention the other literature on the subject, or if he fails to give credit to other workers, this indicates that he is a novice, a dilettante, or an impostor.
5. *Abused Authority.*—If the authorities on the subject are misinterpreted, or misquoted, or if the authority for a view which needs substantiation is unspecified, or if the authority quoted has already been discredited, this should make one suspicious of the author and his ideas.

The ignorant are especially susceptible to appeal to authority, so that where good authority is most needed it is least likely to be found. Francis Bacon pointed out

the character of this fallacy in his comments on the *idols of the theatre*.

There is a very evident tendency to believe that because a man is an authority on one subject, what he has to say on some other subject is therefore equally valuable. Thus it comes about that Henry Ford's views are quoted as evidence for the truth of the doctrine of the transmigration of souls. Thus it happens that the creator of Sherlock Holmes is an authority on the "Fairies." Thus it happens that Sir Oliver Lodge's ability as a physicist lends weight to his words concerning psychic phenomena. The climax is reached when the authority of William Jennings Bryan is invoked to settle the question as to whether evolution is a fact.

In these days of Sunday Supplements and the popularization of science it is necessary to be critical, if one is not to be swamped under a tide of misinformation. Before believing the ideas advocated by any man we must learn how he stands with his fellow scientists. We must also learn what his prejudices are. In the past mistakes have been made in science on the side of too much conservatism. There is a dogmatism of science as well as of religion and politics. But in general scientific scepticism is more defensible than religious dogmatism or political conservatism. A true scientist wants to be convinced, but he is always willing to abandon an old theory or supposed law when the evidence is forthcoming to show him that the old is no longer satisfactory.

Thus by the slow give and take of new evidence permeating and refashioning old theories, science is slowly but surely progressing towards a clearer comprehension of the nature of the world and of man. But this progress is sometimes creeping and painful, and the goal appears endlessly distant. It is very doubtful whether science will ever be able to comprehend completely this entire scheme

of things. And if, in our impatience, we insist upon jumping from the known facts to more or less probable explanations, to comprehensive syntheses of cosmic sweep, we must recognize that at that point we have ceased to function in the realms of science and have passed over the borderline into the domain of metaphysics. But with such excursions, however zestful they may be, we have no concern at present.

Chapter II

THE DISCOVERY AND CLASSIFICATION OF FACTS

SCIENCE is frequently defined as classified knowledge. Let us see what the full implications of this statement are. Knowledge, we have said, is the body of propositions we accept as true. And truth, as ordinarily defined, is correspondence with facts. This therefore brings us to the conclusion that a science is a system of propositions which state the truth about some domain of facts which have been observed. But, it will be noted, these facts are no longer chaotic and unorganized. They are ordered in such a way that uniformities of relation appear between facts, so that invariable laws of behavior can be formulated.

We see, therefore, that the first step in the formulation of a science is the observation of facts. The second step consists in the arranging of the facts thus discovered into a *system*. This is the process of *classifying* facts. But classification is not merely a process of *grouping* facts; it is a process of *orderly* grouping, so that—as we have just said—these facts take on *significant relation* to each other. When these constancies of relation begin to manifest themselves in such a way that they lend themselves to statements in terms of uniform and invariable laws the ideal of science is being approximated.

The goal of science is a system of generalizations, each of which is in the form of the statement, *All A is B*. This type of formulation is ordinarily called a law. A law, from this point of view, is to be regarded as a statement of a uniformity of relation between classes. Thus,

for example, as soon as you say, "All planets revolve about the sun," you have a classification. But this is also the type of induction which is called a law. It is precisely of the character of the generalization, "All material bodies fall in accordance with the law of gravitation." In both cases we have concrete interpretations of the abstract formula, *All A is B.*

In order to prevent confusion we may point out that what we are now saying concerning the formulation and development of a science does not contradict our previous outline of a *complete act of thought.* Neither does it controvert our scheme of the form of a perfect science as outlined earlier in the book, where it was pointed out that in its final form every science would be deductive in character, consisting of a body of concepts (undefined and defined) and propositions (postulates or generalizations) from which the whole body of theorems of that science could be derived. Such would indeed be the case for a perfect science. But in reality no such perfect science exists; and, still further, it is doubtful whether any empirical science will ever fully achieve that status. Certain branches of mathematics (e.g., Euclidian geometry) approximate completeness and perfection, but mathematics as a whole is still imperfect and incomplete, and hence not entirely deductive. And physics, which is as fundamental a natural science as there is, is far from perfect. So that our statement that in a growing science we begin with the observation of facts which are then classified and arranged so as to exhibit laws, uniformities of relation between facts, still remains uncontradicted.

After we have a body of knowledge, a background of beliefs which give us our mental set (or "apperceptive synthesis"), we can investigate any new phenomenon, following then the outlines of our complete act of thought.

If we succeed in understanding or "explaining" the new phenomenon which presents us with a problem, we incorporate the results of this achievement into our body of beliefs which sums up the knowledge embodied in that science. As we will see later on, there is a fundamental and intimate relation between explanation and classification. But for the present we must restrict ourselves to the technique of observation, the first step in the process of the formation of a science of an inductive nature.

The Technique of Observation

Throughout our previous discussion the point has been emphasized that science and fact are organically related. You cannot have science unless you have facts, and the first commandment of scientific method is *respect the facts*. Now many persons think that facts are easy to get. But a knowledge of the history of science proves that facts are frequently elusive things; that frequently supposed facts turn out to be not real facts; and that many genuine facts have not always been recognized as such. While it is true that all facts are known facts, this does not prove that the existence of these facts depends upon their being known.

Science seems to proceed, almost naïvely, on the realistic assumption that facts are what they are, and our knowing or failing to know them makes no fundamental difference to the facts as such. It is for this reason that science has come more and more to realize the necessity for excluding such subjective elements as arise in connection with illusions, personal errors, emotional bias, and the like. The facts are there and can be discovered, if we are willing to take the necessary precautions to avoid distorting those objective facts. Modern science, to a large extent, rests upon the improved technique for get-

ting adequate percepts. This is in part a matter of the invention of more accurate instruments, with finer means of discrimination but it is also in part a matter of learning how to counteract the subjective errors which would otherwise creep into observations and calculations.

To illustrate the statement concerning the difficulty of getting at the facts the following case may be cited. In 1904, Professor Blondlot, a French physicist, announced the discovery of "N-rays." These rays, he said, emanated from all bodies which were capable of storing them. A number of scientists "confirmed" the observations of Blondlot, and it was believed that their demonstrated existence would be of importance in several branches of science. But more careful examination by the sceptical showed that these rays existed in fancy but not in fact. The N-rays owed their supposed existence to errors in observation which had previously escaped notice. More recently A. Nodon, another French scientist, argued that all organic substances of high vitality (e.g., fresh leaves) gave off an emanation which enabled them to take their own photographs. But further experiments suggested the possibility that the results on the photographic plates might be due to the presence of chemical fumes, which had not been excluded. The claims of Nodon are therefore still unverified.

Without going so far as to insist that science and the laboratory are synonymous terms, we must recognize that the growth of science has been closely associated with the laboratories. A laboratory is a place for testing observations. This presupposes the ability to repeat the observations. This means that perceptions can be verified, and facts more definitely established, if we can control the various factors associated with the given phenomenon.

The *object* of an experiment is to test a suspected causal relation. The *method* of procedure consists in

systematically introducing and leaving out the various factors (or *variables*) which have been observed to be associated with the phenomenon (called the *effect*) and noting any corresponding variation in this supposed effect. Experiment, in other words, is producing a phenomenon for the sake of observing it. We may summarize these statements in brief form by employing the terminology of mathematics. A given phenomenon, P, is a function of the variables, a, b, c.

$$P = f\,(a, b, c)$$

As a concrete example of such an experiment where we attempt to establish a causal connection between the phenomenon, P, (the effect) and the cause (A or B or C, or some combination of them), we may suppose that a botanist is investigating the relation between the growth of plants and the incident light which is correlated with plant metabolism. In this case our variables, A, B, C, refer to the different phases of light, and the time of exposure, as follows:

A = the *wave-length,* or the color of the light
B = the *wave-amplitude,* or the intensity of light
C = the *duration* of exposure to the light

By a systematic variation of all these factors according to the various possible combinations of the variables the botanist may be able to set up an equation which summarizes the relation between the phenomenon in question and the accompanying causal factors. This equation is then a causal law which states a uniformity of relation between the several classes of facts.

In many cases where experiments are performed the problem is to *ascertain a simple fact.* But in other cases the problem may be to determine the *variation of a variable.* A third problem may be to determine the *inter-*

relation of variation of two variables. To illustrate: we may suppose that in the first case our problem is to find out how much crime there is in a certain city. Then in the second case our problem will be to determine how this number of crimes varies from year to year. A third problem would be to determine the inter-relation of the variation of crime and the variation of poverty for the same years.

As we increase the complexity of a situation having its effect (or effects) and its appropriate cause (or causes) we increase the difficulty of interpreting whatever results of observation we may secure. This is so in part because the more complex the situation the more numerous are the variables and the harder it becomes to exclude errors of one sort or another. The types of errors which can creep in have been pretty well standardized as to their nature, and the following are the conventional types of errors which must be allowed for in interpreting experimental results.

Errors in perception are due to—

1. Unfavorable physical conditions
2. Unfavorable physiological conditions
3. Unfavorable psychological conditions

I. Unfavorable physical conditions may arise in connection with the type of instrument employed. A certain amount of error is always introduced through the use of apparatus. Contrivances which depend upon light for their use and electrical recording devices (voltmeters, etc.) are not accurate beyond certain limits. In some cases it is therefore futile to try for accuracy beyond certain limits. The physical media in which, or through which, the observations are made and transmitted will also sometimes give rise to errors, distortions, etc. The stick which looks bent when partly immersed in water

is an illustration of this. The "fixing" of biological specimens by staining in order to make certain structures visible might also be so regarded.

II. In order to get accurate percepts the sense organs which are employed must be normal and the individual as a whole be in fit condition. Unfavorable physiological conditions, such as are produced by the influence of drugs, fatigue, and alcohol, can be avoided. But there are other physiological limitations to perception which we cannot escape.

We are so constructed by nature that we can perceive some things and not others. On the one hand, some events happen so quickly that we cannot perceive them. We cannot see the spokes of a wheel which is revolving at a speed above the critical velocity for such perception. In order to perceive a light sensation the sensation must endure for a minimal period of about $\frac{1}{16}$th of a second. Sleight of hand tricks take advantage of this limitation of the visual apparatus. On the other hand, some events happen too slowly to be perceived as such. The movements of the shadow on a sundial, the growth of plants, and certain movements of heavenly bodies are not ordinarily perceptible.

The first type of limitation, the inability to perceive events which are too rapid for the eye, can be put to practical use. The illusion of the "moving picture" and the continuity of sound of a vibrating body whose frequency is above a dozen vibrations per second are made the basis of very pleasing experiences. The term "physiological inertia" has been suggested as a label for this fact that it takes a small interval of time to get a sense organ to function, which then continues to function for a small fraction of a second after the objective stimulus has ceased. This fact is part of the phenomenon of the personal "reaction time" of human beings. Indeed, the

science of psychology (especially *psychophysics*) took its origin from this discovery that the reaction time varies for different individuals.

III. Psychological sources of error enter through the force of habit, the emotional predispositions of hope or fear, and, in general, what is called "mental set." Misperception (or malobservation) consists for the most part in mistaking inferences for facts of observation. This error is common and difficult to eradicate precisely because of the easy transition from sensation → perception → judgment. One may make mistakes through casual inattention, careless observation, want of practice in a science, or from not knowing what to look for. A person may think he observes what he hopes to find or fears to find. He may add or leave out, as his bias, emotions, or habits dictate.

This tendency of the mind to objectify the subjective is so strong in some persons that they become the victims of obsessions. Such persons, since they are in the minority, are said to be abnormal. As the unfortunate victims of delusions or hallucinations (judged by the social test) they are put into institutions for the insane. But even large groups may as a whole become subject to social hallucinations or delusions. Epidemics of misperception and false belief have at times swept across a people like a tidal wave. The "Angels of Mons" of the late war would probably be regarded by most critical persons as an illustration of such group self-hypnosis. It has also been suggested by some sceptics that the alleged miracles which fill the annals of most religions of the world are instances of collective hallucinations, induced by excessive emotional set. There is also reason to believe that many of the alleged phenomena of psychical research may be explained away as being due to a tendency to perceive what you want to perceive.

Because of the special conditions peculiar to each science it usually requires years of study and practise before one can claim to be an expert in that science. However, all of us employ in a crude way the test which trained scientists employ in their more refined technique. It is in this sense that it may be said that science is simply refined common sense. We all know that if there is any doubt about the reality of a certain apparently real phenomenon we appeal to others, we check up one sense organ against another, we try to repeat the observation, and so on.

If we are certain that the physical, physiological and psychological conditions are favorable to adequate percepts; if we have employed all possible checks; if we have made due allowances for whatever unavoidable errors there may have crept in; if our observations cohere consistently with each other, and agree with the observations of other reliable investigators;—if we have taken all these precautions then we are justified in accepting as valid or real the phenomenon which we have previously accepted tentatively as a possibly real object of perception.

Classification in Science

The result of reliable observation is the accumulation of facts. But uncoördinated facts are of little value. The problem is to assemble these facts so that significant relations between them begin to appear. If we had to remember every fact as a fact, the mental burden would be too heavy for us to carry. So we adopt principles of economy of mental effort.[1] Classification is one of the most useful ways of ordering knowledge in systematic form. If we can place some new entity in a class we

[1] For a statement of the notion of science as an economy of thought see Ernst Mach's essay "On the Economical Nature of Physical Inquiry," in his *Popular Scientific Lectures.*

immediately know more about it than we would if we had to deal with it as an isolated or unclassified thing. We know the properties of things as instances of their proper classes, and this also tells us about things in terms of their relations to other classes.

Classification is the intermediate link between induction and deduction. The placing of an object in a class is a process of induction. If you say "all metals are elements" you have classified metals as a logical species of the genus elements. But you have also made an inductive generalization. And such generalizations in turn constitute the premises of deductive arguments. The propositions of syllogisms consist of statements of the relations of classes to each other. But the original formation of these classes is a process of passing from particulars to a generalization, and it is for that reason that classification is dealt with as a phase of inductive logic.[2]

After we have discovered the facts we must next adopt the type of classification which will best enable us to use this knowledge. Broadly speaking, there are two kinds of classification: *artificial* and *natural.* The contrasts between these two types are important, so that we must dwell upon this matter at some length.

In every case where we have a classification the principle of classification (the *principium divisionis*) rests upon something in the nature of the things to be classified. In no case is the principle of division purely arbitrary or subjective. There are always objective differences which justify the grouping. The difference between an artificial and a natural classification is not to be sought in the sub-

[2] As various writers have noted, there is a close connection between classification and definition. To define is to classify, to place the entity designated by the term to be defined in its logical genus and species. This intimate connection also appears in symbolic logic. The *concept* to be defined is a propositional function, with one variable, and a *class* is all the objects satisfying some propositional function.

jectivity or the objectivity of the qualities which are selected as the basis of the classification.

Artificial Classification.—In an artificial classification a property is selected which serves some temporary or restricted purpose. Thus, to illustrate, one might classify flowers or books in a library on the basis of their color, putting all flowers or books of one color in one group, and so on. This would be an artificial classification. Artificial classifications serve some temporary purpose; or it may be a fairly permanent purpose restricted to a single individual. The esthetic appearance of a collection of books is ordinarily an unessential or accidental (though real) characteristic. It serves no useful purposes in the long run to know the colors of things, since color is such a variable quality.

Only where color is a *diagnostic quality* does it tell us anything about the essential nature of the things classified according to color. For example, among birds, it is a fairly general rule that the male of the species has the brighter plumage (thus reversing the ordinary human relation). In such cases one may be able to judge the sex of a bird by noting the character of its plumage. Then one knows something more about the creature in addition to its color. The same is true of chemical solutions, where color is sometimes a diagnostic quality. But in these cases, where color is a symptom of other correlated qualities essential to the nature of the things themselves, a color classification is well on the road to a natural classification.

Natural Classification.—The purpose of a natural classification is to summarize and make available for future purposes the knowledge which we have obtained in the past. This knowledge must be so organized as to serve the widest number of human purposes over the longest period of time. Hence an unessential or acci-

dental characteristic will not serve as the basis for a natural classification, since permanent purposes must be related to the enduring features of things. We need, therefore, as the basis of a natural classification some quality which is fundamental to the things to be classified.

As Jevons,[3] following others, has pointed out, the most important prerequisite of a good classification is that *it shall enable the greatest possible number of general assertions to be made.* In order to make this possible the *guiding principle* in the formation of a natural classification is to *select properties or qualities having the widest group of correlated properties invariably associated with them.* One of the best illustrations of this is to be found in the type of classification now employed in the biological sciences. For example, it is no longer acceptable to classify plants on the basis of the number of stamens they possess (the old classification of Linnæus); some more fundamental quality must be employed.

Shape, color, and other superficial differences among living forms will not serve as the basis for classifying plants or animals. The difference between vertebrates and invertebrates is not a difference in size or shape primarily, but a difference in anatomical structure and derivation which has correlated with it other essential differences in mode of life. As A. D. Ritchie[4] puts it: "In the biological sciences the expressed aim in classification is to display the actual blood relationship between animals and plants and not merely to catalogue them in a convenient way. Thus if the classification is correctly done it will display the laws of heredity and development when these come to be discovered."

[3] *Elementary Lessons in Logic,* by W. Stanley Jevons, 1886, p. 280.
[4] *Scientific Method,* p. 47.

The following are the requirements of a good classification:

1. Every classification should be made on the basis of differences appearing in all the members of the entire group to be classified.
2. Only one principle of division into classes should be employed.
3. Constituent species must be mutually exclusive; there must be no overlapping of groups.
4. The classification must be exhaustive; the constituent species must equal the entire class when added together.[5]

Many investigators regard classification of facts as part of the preliminary spade work in the construction of science, looking upon explanation as the later and higher goal towards which the developing science moves. However, there are good reasons for believing that much that goes under the name "explanation" is nothing other than classification. But to develop this idea requires a separate discussion which we must reserve for a later chapter.

[5] If we follow the principles of symbolic logic and regard classification as a special case of the expansion of complex terms it is possible to employ several principles of division and still make the classification exhaustive. But in order to do this we must be familiar with Boole's Law. For a discussion of this method of treatment see A. E. Avey's *The Function and Forms of Thought,* Ch. XI.

Chapter III

THE RÔLE OF HYPOTHESES IN SCIENCE

IN order to keep our intellectual bearings let us recall our outline of the act of thought. As they were previously enunciated, the stages in a complete movement of reasoning are:

I. The appearance of a problem.
II. The rise of a suggestion—a tentative explanation, or solution.
III. Deductions from the hypothesis.
IV. Looking for the predicted facts.
V. Verification—the establishing of a law.

What has been stated in the previous two chapters is concerned with the building up of a background of knowledge which is necessary in order to have a problem. Unthinking and inanimate things, having no background of experience or accumulated knowledge, cannot have problems and cannot think. And thinking is the tackling of new situations in the light of previous experiences. The stored knowledge gained from these experiences, as we have just seen, is embodied in the classifications which we employ. Now it is our problem to observe how the second step, the formulation of a suggestion or an hypothesis, fits into the movement of thought. Our discussion will follow this outline: (1) What is an hypothesis? (2) The values of an hypothesis. (3) How do hypotheses originate? (4) The method of plural hypotheses. (5) The requirements of a good hypothesis.

What Is an Hypothesis?—An hypothesis is a guess, or

a supposition. It is something which is entertained as possibly true because something else is believed to be true. As a tentative clue this hypothetical product of reasoning needs to be tested. An hypothesis is any conjecture which is suggested as a possibility for verification.

Science, we have observed, is restricted to that which is perceptible or capable of being inferred from the perceptible world. From this point of view an *hypothesis* is the process of ideally anticipating further perceptions by mentally visualizing those images which are logically implied by the perceptions already given. *Verification* is in part the process of reducing those mental images to perceptions; that is, of finding in the real world the correspondents to the ideal images implied by the original experiences which gave rise to the problem, and which stand in need of "explanation." A true hypothesis, therefore, is a fact in the making; and a verified hypothesis terminates in factuality.

The Values of an Hypothesis.—The functions of an hypothesis are several. In the first place, an hypothesis is useful in tying facts together. Like a good classification, an hypothesis relates and organizes facts. Indeed, a classification is simply an hypothesis which has been substantiated through repeated usage. In chemistry the atomic theory (formerly an hypothesis) gives an explanation of the vast mass of facts concerning chemical affinity, valence, the laws of the periodic table, etc. It ties these facts together by placing them in classes where uniformities of behavior obtain. In biology the doctrine of evolution "explains" a vast mass of facts through the ordering of these facts in a naturally systematic form. If an hypothesis does not bring order out of chaos by displaying significant relations between the groups of facts it was devised to fit it is not worth much.

The second function or value of an hypothesis is to

suggest experiment. The process of forming an hypothesis carries with it the anticipation of further experiences. But this anticipation stands in need of verification. The experiments which are performed to test the hypothesis consist in looking for the further facts which ought to be found if the hypothesis is true. If all the facts which are predicted by the experiment are discovered, and if no antagonistic facts are revealed, the hypothesis gains greatly in plausibility. The ultimate goal of an hypothesis is complete verification, but it is doubtful whether any empirical science can lay claim to absolute certainty. At the close of this chapter we will return to this matter.

How Do Hypotheses Originate?—Hypotheses are the products of the creative imagination. The term imagination has several connotations. Literally, imagination is the use of mental images. But if one is termed an "imaginative" person this usually means that he has a tendency to ignore realities, to fly into the realm of impractical unreality. In the sense in which the term imagination is here used, as designating the capacity to devise hypotheses, it refers to constructiveness on the basis of experience. Nothing is more useful to an original scientific investigator than a *disciplined* imagination, a fact which has been eloquently expounded by John Tyndall in his well-known essay on "The Scientific Use of the Imagination."

And now how do we get these hypotheses which are to explain the facts of a mystery? Is there a uniform and lawful method in this, or are hypotheses plucked from the ether, so to speak, by some mysterious and intuitive process which defies analysis? While it is true that one cannot give a set of rules which will make a scientific genius out of a moron, nevertheless there are certain modes of procedure which can be employed by anyone with sufficient ingenuity to profit by the rules.

(A) In the following chapter on the establishment of causal laws, we will see that all suspected causal relations are first suggested by observing one thing to happen along with, or shortly after, another. This method is termed the method of agreement. We cannot now discuss this method, but can only point out that until a supposed causal law is verified it must be considered as an hypothesis. Such laws are sometimes not considered to give us explanations; nevertheless, such generalizations are hypotheticals. One way, therefore, of getting hypotheses is to observe uniform connections of events.

(B) A second way of getting hypotheses is to take generalizations already established and see whether they may not be turned around. This is making use of the conversion of propositions. Some processes and events of nature are reversible. While temporal relations are usually one-way (or irreversible), causal relations are sometimes two-way (or reversible). In some cases it is possible to infer that if $A \rightarrow B$, then $B \rightarrow A$. An interesting illustration of this arises in connection with the relation between electricity and magnetism. If a moving electric current generates a magnetic field, will cutting the lines of force of a magnetic field generate an electric current? The effective functioning of the dynamo gives us the answer in the affirmative.

Recently Sir Oliver Lodge raised the question: If the stopping of a rapidly moving electron generates light rays, will the stopping of light rays generate an electron? As yet this problem of the creation of matter from radiation by a process which is the reverse of the conversion of matter into radiant energy has not yielded to investigation. But the illustration gives us a fascinating application of this second method of discovering hypotheses.

(C) The third source of hypotheses is through the use of argument by analogy. This type of reasoning is

one of the most fruitful modes of thought which we have. But like many other useful things, it is also one of the most dangerous when not properly employed. This is illustrated in the case of primitive thought in the type of explanation known as *animism*. The tendency to personify the forces of nature, to interpret all phenomena as due to the purposes of some indwelling spirits or vital forces, is clearly a result of argument by analogy.

Argument by analogy is really after the pattern of a proportion, and in abstract form appears as follows:

$$a:b::c:d$$

In the case of animism the interpretation of this is:

Human purpose: *behavior*: : *spirits*: *natural phenomena*

The defect of primitive thought did not consist in resorting to argument by analogy, but in employing the wrong sorts of analogies. Whenever an inference is made through the use of analogical reasoning one must be sure that the points of similarity are more significant than the points of difference. It is not merely a matter of counting up the points of similarity and difference and then equating the net results. Two things may be alike in all respects except one, and yet this single point of difference may be sufficient to upset the analogy. This shows that we must weigh the value of each similarity and difference and then balance the accounts.[1]

When rightly employed reasoning by analogy is fertile and productive. A scientist generally attacks his new problems by working back into old experiences to find something analogous to the present situation. In so far as a new problem resembles an old problem to that extent the new solution will resemble the old solution. This is—

New problem: *old problem*: : *new solution*: *old solution*

[1] False analogies are discussed on page 249.

As a specific illustration of this we may take the doctrine which was presented as a solution to the problem of the nature of light. When such facts as the interference of light were discovered, it was argued by analogy that if there are waves there must be a medium in which these waves occur and which transmits these waves from place to place. Hence the ether of space or the luminiferous ether was created as the medium in which waves were transmitted. The analogy here is this:

Ether waves: *ether*:: *water waves*: *water*

Now there are serious reasons for doubting whether this is a good analogy. It is possible that in the case of light there may be waves without there being something (a medium) in which these undulations occur.

Many causal relations can be put in the form of an analogy. If fire is the cause of heat, we may set down the proportion:

$$\frac{fire}{heat} = \frac{cause}{effect}$$

Reverting back to our statement that a natural law is really an established correlation between classes, we can see why it is possible to state causal relations in the form of an analogy. Analogy is essentially a classification in the making. If one thing becomes sufficiently analogous to another so that it duplicates all the properties and forms of behavior of the other the two become identical. This is what is meant by the phrase, the *identity of indiscernibles*. Theoretically identity may be regarded as the limiting case of analogy, as two things become more and more alike.

It is doubtful whether two members of the same class, even though they are sufficiently alike to be labelled by the same class name, are ever absolutely identical in every

respect. There is no evidence, for example, that each electron is exactly like every other electron; but they are sufficiently alike in mass, dimensions, and behavior so that whatever differences there may be are negligible for present purposes. But in the future these apparently insignificant differences (if such there be) may become of importance. These remarks have an important bearing on the nature of explanation, but we must take that up later on.

In order to illustrate the extensive use of analogical reasoning, and to suggest examples for further study, the following illustrations may be suggested:

(a) The organismic theory of society.
(b) Is Mars inhabited?
(c) Darwin's theory of evolution and Malthus' law of population.
(d) The "argument from design"—the "watchmaker" theory of the universe.
(e) Habit formation and various physical analogies.
(f) Freudian psychology and hydrodynamics.

Summing up the substance of our argument thus far we may say (*a*) that reasoning by analogy can never prove anything, but (*b*) that analogies are very useful in suggesting hypotheses; and (*c*) that most supposed identities are really close analogies, though (*d*) for practical purposes second order differences are usually unimportant, if the first order similarities between the elements of a class correspond point for point.

The Method of Plural Hypotheses.—The method of plural hypotheses tells us that after we observe a phenomenon which requires explanation we should formulate all possible hypotheses which might conceivably elucidate the mystery. The next step is then to decide between the rival hypotheses. Various hypotheses are tested by making deductions from them and then looking for the additional facts which are predicted by the hypothesis.

It must be remembered, however, that no hypothesis is completely proved by producing evidence for it. It must be shown that the accepted hypothesis is the *only* one which satisfactorily explains the facts in the case. That is to say, it must be shown that the other possible hypotheses do not explain all the facts as satisfactorily as the accepted hypothesis. Not only must it be shown that if the hypothesis is true the facts will be as found, but it must also be shown that if the hypothesis is not true the facts will not be as found. In some cases it is possible to formulate a crucial experiment (*experimentum crucis*) which will decide unequivocally between rival hypotheses, but such cases do not happen often.

The balancing of evidence for and against an hypothesis introduces the concept of *probability*. If the evidence converges to support one hypothesis rather than another the hypothesis takes on a greater degree of certainty or truth-value. In this world we very rarely get theoretical certainty. But we do get practical certainty. That is, the probability of the truth of certain beliefs becomes sufficiently great to make any other supposition extremely improbable. Even *circumstantial evidence* may become sufficiently strong to make the hypothesis that a certain individual committed a given murder practically certain, in which case a jury may be willing to condemn a man to death on its basis.

It sometimes happens that two rival hypotheses equally well explain the facts, so far as we are able to see. In such cases what are we to do? The answer is that if two hypotheses equally well explain the facts we should choose the simpler hypothesis. This decision is in accordance with the law of parsimony, which tells us that nature prefers simplicity. As we have already seen, in its original form this doctrine is known as Occam's razor.

This principle is frequently legitimately appealed to,

but it is also sometimes abused. In the first place, we cannot always be sure that nature *does* prefer simplicity or take the "line of least resistance." In the second place, there is the problem of determining which after all is the simpler hypothesis. Presumably economy of mental effort is the test of simplicity, but how shall we measure this? It is interesting to note that both the systems of Newtonian physics and the relativity physics of Einstein have been defended on the grounds of their greater simplicity, so that the problem of simplicity is not as simple as it seems. Anything will appear to be "simple" once you understand it, and "understanding" a phenomenon is frequently only a matter of becoming familiar with it. The "miracles" of yesterday are the commonplaces of today largely because we are now accustomed to them.

Finally, in connection with the method of plural hypotheses, we must observe that it occasionally happens that several rival hypotheses are equally simple, and, so far as we can see, apparently explain the facts equally well. In such cases it is well to suspend judgment. This is hard to do, for we are first of all creatures of action and accordingly do not like indecision. But there are many cases where suspended judgment is called for. The evidence is not all in, and we don't yet know which of the several possible hypotheses is the correct one.

At present the wave theory of light and the corpuscular theory are both in the balance. It is sometimes possible to reconcile two hypotheses under some broader generalization which then includes the apparently conflicting hypotheses as corollaries. Perhaps this will be the solution to the problem of the nature of light. In biology the doctrine of the transmission of acquired characteristics and the opposite view, sometimes called the doctrine of the continuity of the germ plasm, are still fighting for supremacy. In the same way, the hereditarians and

the environmentalists are both claiming that the evidence bears out the truth of their respective doctrines as applied to human beings. In all such cases, where the experts are not agreed, it is advisable for the layman to hold his peace. This is an attitude which should be cultivated. Where there is not sufficient evidence of an unequivocal character to decide the issue we should be honest enough to admit that we do not yet know what the answer to the problem is.

A really great thinker is one who is not only fertile at devising ingenious hypotheses, but—what is frequently much more difficult—is also clever at constructing tests to verify his speculations. This critical attitude towards one's own ideas is commended by Faraday in the following familiar lines:

"The world little knows how many thoughts and theories which have passed through the mind of a scientific investigator have been crushed in silence and secrecy by his own severe criticism and adverse examination; that in the most successful instances not a tenth of the suggestions, the hopes, the wishes, the preliminary conclusions have been realized."

The Requirements of a Good Hypothesis.—Before an hypothesis can be regarded as firmly established it must run the gauntlet of the following requirements:

I. *The hypothesis must be plausible.*—It must be remembered, however, that there is a certain relativity about plausibility and conceivability. An hypothesis which is plausible or implausible in one age, or for one group, may become the precise opposite in some other age or in the light of new evidence. We must therefore qualify this statement to read: plausible in the light of existing knowledge.

II. *The hypothesis must be adequate.*—It must explain *all* the facts of the problem. But sometimes a phenome-

non is complex, and one hypothesis explains some of the facts and another hypothesis explains the remainder of the facts. In such cases we must either develop a compromise hypothesis, or break up the problem into two relatively independent sets of phenomena, each requiring its own appropriate explanation.

III. *The hypothesis should contain no internal contradictions.*—In some cases this matter is easily settled, and in other cases the test is more difficult to apply.

IV. *The hypothesis should be as simple as possible.*—We have already discussed this test.

V. *The hypothesis must be capable of proof or disproof.*—If a scientist should postulate as the cause for a certain phenomenon of our solar system the existence of some peculiar condition (a mountain made of iron, let us say) on the other side of the moon, which no creature on earth has ever seen, this would not be a satisfactory explanation. To be sure, no one could disprove the hypothesis with absolute certainty, but that is hardly adequate grounds for believing in an idea. When a man asserts an explanation which is not in accordance with ordinary experience the burden of proof rests upon him.

In connection with this rule we must remember that hypotheses which are incapable of proof in one age may become susceptible of verification in a later age. Not so very long ago it was asserted that we would never know the chemical constitution of the stars, but the invention of the spectroscope soon upset that prediction.

In the following section we will discuss one of the most remarkable verifications of an hypothesis which the history of science affords.

Verification and the Prediction of Facts

The way to test an hypothesis is to make deductions from it; that is, to infer the additional facts which are

implied by that hypothesis. We have already discussed the methods of deductive reasoning by means of which the implications of the facts stated in the accepted premises are elaborated. Now we need to relate this principle to a concrete example.

If it were possible to treat it in terms which were intelligible to the intelligent layman, one of the most constructive movements of scientific thought which might be studied for logical purposes is to be found in the theory of relativity of Albert Einstein. For here the way in which experiment and hypothesis play into each other is exemplified in an unparalleled degree. Since the theory of the doctrine cannot be presented, the reader will have to take the word of the experts that the following experiments are indeed genuine deductions from the theory:

(*a*) The bending of the light which is passing close to the sun on its way to terrestrial observers. This bending is due to the non-Euclidian character of space-time and the equivalence of mass and energy (light).

(*b*) The shift in the Fraunhofer lines in the solar spectrum towards the red end, due to the fact that in the theory of Einstein an atom in a centrifugal field of force ought to vibrate more slowly.

The number of tests of the theory of relativity depends upon what you are going to call a "test." In a sense the negative results of the Michelson-Morley experiments and the advance in the perihelion of the orbit of the planet Mercury may also be regarded as "tests" of the relativity theory. In addition, there are phenomena in the realm of atomic physics which seem to support the theory, as Dr. R. A. Millikan has pointed out in his book on *The Electron* (2nd. Ed., p. 237 and p. 256).

For the purposes of our analysis only one test need be examined. In the case of the deflection of light the evidence is positive and unambiguous, and accordingly

has strengthened the truth-value of the hypothesis enormously. But from a theoretical point of view no amount of experimental evidence can establish the relativity theory as true beyond the shadow of a doubt. To illustrate this let us put the test in the form of an hypothetical proposition. The doctrine says this: *If the theory of relativity is true, light will be deflected so much when passing near the sun.* In abstract form this is our old proposition: If *A is B, then C is D.* But, as we now know, affirming the consequent ("Light is deflected") does not permit us to affirm the antecedent ("Therefore the theory is true") in a hypothetical syllogism.

Before the theory of relativity can be accepted as true it must be shown that this is the *only* theory which can explain *all* the facts. Moreover, as Dr. Harry T. Costello has pointed out to the author, when you discover the facts predicted by an hypothesis you do not verify the hypothesis, you verify the results deduced from the hypothesis. And you still need to determine how favorable you have rendered the hypothesis itself.

It is true that at present no other theory offers itself as an adequate explanation of the same facts, but it cannot be said that in the future such a candidate will not appear. Einstein himself, like all really great thinkers, is quite objective towards his own theory. As he says, "No amount of experimentation can ever prove me right; a single experiment may at any time prove me wrong."

Bringing this discussion of the rôle of hypothesis to a conclusion, we may sum up as follows: If you find that a given hypothesis explains a phenomenon (the set of facts presenting the problem); that the deduction made from the hypothesis is discovered by further observation; and that no other hypothesis equally well explains the facts,—then the hypothesis passes into the status of a theory, eventually, perhaps, to become a law.

Chapter IV

THE ESTABLISHING OF CAUSAL LAWS

A CONSIDERATION of the subject of causality calls to our attention another illustration of the thesis that the most useful tools of thought and action are frequently the most potent sources of trouble. The notion of causality is one of the most practically useful concepts the human race possesses, and yet from a theoretical point of view it is one of the most elusive concepts thinkers have ever tried to analyze. Because of this it would be well to postpone our theoretical discussion until we have considered the uses of this tool of adjustment.

It is necessary, however, to formulate a provisional working definition of the term. This can be done without a digression into theory. The terms cause and effect are correlative terms; like the terms parent and child, they go together. Accordingly, for our purposes the cause of anything may be said to be that without which the effect would not occur; and the effect may be said to be that which depends upon the cause of its existence. From this point of view the pragmatic test of causality is this: *What difference does it make* to the phenomenon in which we are interested (the "effect"), when we introduce and leave out the factor which is supposed to be causally connected with the phenomenon?

Defining the cause in this way permits us to draw conclusions which are important for the determination of causal relations. These conclusions are as follows: (1) That cannot be the cause in the absence of which the

phenomenon occurs. (2) That is not the cause in the presence of which the phenomenon fails to occur. (3) That is not the cause which varies when the phenomenon is constant, or is constant when the phenomenon varies.

These three statements give us the essence of what are known as Mill's canons of induction. These canons are named after John Stuart Mill, their formulator, who carried the work begun by Francis Bacon and Sir John Hershel to a further stage of development. Since the time of Mill those who have given his statements of the methods have felt called upon to rephrase the wording. Mill's methods are designated as follows: (1) The method of agreement; (2) the method of difference; (3) the joint method; (4) the method of concomitant variation; and (5) the method of residues.

Let us discuss each of these methods in the order given.

The *Method of Agreement.*—Most suspected causal relations are suggested to the investigator through the observation that two things are constantly happening together (or one shortly after the other). The possibility then presents itself that the thing which precedes is the cause of the phenomenon which succeeds, the effect. In Mill's own words: "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon."

This may be summed up in symbolic form as follows:

$$\begin{array}{l} A\ B\ C \rightarrow R \\ A\ D\ E \rightarrow R \\ A\ F\ G \rightarrow R \end{array}$$

The history of science is replete with examples of the use of this method. One illustration of this is to be found in the experiment whereby Sir David Brewster

established the fact that the substance, mother-of-pearl, has nothing to do with the brilliant colors associated with this material. Like many other discoveries in science, this was a result of an accident. Sir David took a wax impression of mother-of-pearl and found, to his surprise, that colors were also associated with this surface. He then tried out this experiment with other substances, and found that in each case the colors were present. Since the only factor these substances had in common was the *form* which reproduced the impression of the mother-of-pearl, the inference was drawn that the form was the cause for the display of colors.

As a homelier example of the use of this method let us suppose that a man has been having a headache almost every day for some time. If observation revealed the fact that on the days on which he had the headache he had drunk coffee for breakfast, and this was the only circumstance invariably correlated with the effect, by the method of agreement the man would infer that the drinking of the coffee was the cause of the headache.

Now we must observe some of the precautions which must be taken in employing this method. While it is true that most causal relations are suggested by observing things to happen closely connected in a temporal sequence, *the method of agreement never proves conclusively the truth of the suspected causal relation.* You must always check the possible causal law by the use of one or more of the other methods. The reasons why the method of agreement, when used alone, never is sufficient, are as follows:

In the first place, two things may be associated together on all the occasions covered by our observation *as a matter of coincidence.* All superstitions are probably a result of the fact that someone noted one thing to be associated with another, and therefore concluded that the

two were causally connected. In some cases, the negative instances which would discredit the suspected causal connection were not noticed, because the originator of the idea is not interested in finding this evidence. Even scientists are endowed with the human tendency to favor the offspring of their own intellects, so that the evidence which would upset their hypotheses is often ignored or overlooked. In other cases the negative instances may be absent, so that even with the best of intentions an investigator will not be able to discover any. The following story illustrates the point. A number of Greek sailors (this was in the good old days of ancient Athens) were gathered together in a port telling stories. It became manifest that each of them had at some time or other been on the high seas when a severe storm threatened to sink the ship. All of them had prayed to the gods to save them; and here they were, all alive and safe. Out of gratitude to the god of the sea for having answered their prayers they proposed to take up a collection and use the funds to erect a monument to Neptune. But before they could get any further one old sceptic saw the fallacy, and said: "Before you go further I would like to hear from those who prayed to Neptune and then went to the bottom of the sea." In this case the negative instances were not there to tell their side of the story.

Primitive man was much victimized by the logical fallacy of utter trust in the use of the method of agreement. The doctrine that if one thing happens after another it therefore happens on account of it (*post hoc ergo propter hoc*) is now called the *fallacy of false cause.*

A second reason why this method is not infallible lies in the possibility that the so-called cause and effect, constantly occurring together, are both the result of a more ultimate (or less obvious) cause. For example, there is

a correlation between the aurora and electrical disturbances on the earth (as manifested in disturbances in telegraph transmission, compasses, and radio reception). By the method of agreement we might infer that the auroral activity is the cause of disturbed electrical conditions on the earth, but the real fact is that both are the effects of electrical disturbances on the sun, manifesting themselves in increased sun-spot activity. Many other illustrations of the same fact can be suggested.

A third limitation of this method arises in connection with what is called the "plurality of causes." This term refers to the fact that a given effect may have one cause on one occasion and another cause on some other occasion. Thus, referring back to our example of the cause of the headache, it may be that on one occasion the cause was smoking and on another occasion the cause was the drinking of coffee. In other cases a "collocation of causes" might have produced the effect. But the method of agreement insists on finding one isolated cause invariably associated with the effect. To escape this difficulty one may postulate that if we have a headache due to smoking and another due to drinking we do not have a genuine case of plurality of causes. Further refinement will show that we have only one cause for one effect. Just as we can break up the cause, so we can break up the effect. Hence we can say there are as many types of headaches as there are types of causes of headaches; in which case the method of agreement can still be employed, though our analysis will now have to be of a more refined sort. This, at any rate, is what happens when we reduce the various causes of heat (friction, combustion, electrical resistance, etc.) to one type of causes, namely, increased molecular motion, and abolish what appears to be a real case of "plurality of causes."

The *Method of Difference*.—Our previous discussion

has made it clear that the method of agreement needs to be supplemented. One way to provide a check is to employ the method of difference. Mill's statement of this method is as follows: "If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect, or cause, or a necessary part of the cause, of the phenomenon."

The symbolic form for this method is:

A B C D R
—B C D —

It will be noted that while the method of agreement varies all but one circumstance, the method of difference varies only one circumstance and keeps every other factor the same.

As a concrete example of the use of this method consider the following. If two objects which are of similar chemical composition, character of surface, etc., differed only in one respect, namely, that one was cooler than the surrounding atmosphere, while the other was not, and if dew were deposited on the first object and not on the second, we would conclude that the only circumstance in which the two differed was the cause of the phenomenon in question. That is, the fact that the first object was cooler than the surrounding atmosphere, whereas the second was not, while dew is deposited on the first and not on the second, suggests that the difference in temperature of the two objects is responsible for the presence of dew, or is at least causally connected with the phenomenon as an essential condition for the effect.

Many laboratory experiments depend on the use of the method of difference. To be sure, it is frequently hard to

realize in detail the conditions presupposed by the use of this method. It is seldom that two circumstances will differ in only one respect. But we try to get cases where there is only one *significant* difference. The insignificant differences (such as mere difference of spatial position) are not supposed to make any difference to the result. Thus, when carefully employed, this provides a reliable check. Most laboratory experiments have "controls," which duplicate so far as is possible the ideal conditions demanded by this method. It is especially difficult to achieve these conditions in experiments where human beings are the subjects. It is for this reason that experiments in the biological and social sciences are so difficult to perform and interpret. At the present time the social sciences are marked by the almost complete absence of "controls."

The *Joint Method.*—It is clear that one cannot note the absence of a certain factor—the possible cause—unless one is looking to see whether the factor will or will not be present. And the reason the investigator is on the lookout is because he has been led, by the use of the method of agreement, to suspect that a certain factor is a possible cause of the phenomenon. In other words, one does not employ the method of difference without having also employed the method of agreement. And when the two, the method of agreement and the method of difference, are employed together, we have the joint method.

Mill's statement of the joint method is as follows: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the

cause of the phenomenon." This statement of Mill means this: The experimenter must collect instances where the known effect is present and compare these cases to see wherein they agree. He also collects instances where the effect is absent, while yet the various possible causes are present. He then compares the two sets of instances to see wherein they differ. Thus a group comparison is made.

The symbolic form for this method is:

If A B C D → R
A X Y D → R

then both A and D appear to be causally related to R.

But if A M N O ⟶ —
A X C B ⟶ —

then not A but D is causally related to R.

As a concrete example of the use of this method let us suppose that in a certain town there is an epidemic of typhoid fever, and our problem is to find the cause of the epidemic, i.e., the source of the germs. If an investigation showed that in all cases where typhoid was present the only common factor in these cases was the use of milk from a certain dairy company, while in the other cases where the disease was absent these families used milk from other companies, while other circumstances (source of food, water, etc.) were the same when compared with instances where the effect was present, we would infer that the source of the disease germs was to be found in the milk supplied by this certain dairy.

In cases similar to the one we have just described the real situation is not likely to be as simple as we have portrayed it. In the case of infectious diseases it is not probable that there will be a point-for-point correspondence between the presence of the cause and the presence of the effect, and the absence of the cause and the absence of the effect. Such things as "immunity," "counteracting

causes," and the appearance of new cases as a result of contagion will upset a perfect correlation. But if there is a statistical correlation this will be sufficient to suggest the probable cause, which can then be investigated in a more refined way.

The *Method of Concomitant Variation.*—This method is useful when the factors in the supposed cause cannot be completely left out, as the method of difference demands. Such phenomena as heat, air pressure, and the like, cannot be introduced and left out absolutely and at will, though they can be varied in amount or intensity within certain maximal and minimal limits. In such cases we may conclude, as Mill tells us, that "whatever phenomena varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation." In symbolic form this reduces to the following:

$$\begin{aligned} X &\rightarrow Y \\ 2X &\rightarrow 2Y \\ 3X &\rightarrow 3Y \\ \tfrac{1}{2}X &\rightarrow \tfrac{1}{2}Y \end{aligned}$$

If one is investigating the cause (medium) for the transmission of sound waves he may use a bell jar with a vacuum pump attached to exhaust the contained atmosphere. If an electric bell is wired up inside the jar, with the button on the outside, the ringing of the bell inside the jar can be heard when the button is pushed. Starting with an air pressure inside the jar which is equal to that on the outside, the atmosphere can be exhausted more and more. As we approach closer and closer to a vacuum the ringing becomes fainter and fainter, until the sound can no longer be discriminated by the ear, even though we do not achieve a perfect vacuum. Here there is a quantitative correlation between the variation of the

cause (the presence of an atmosphere) and the variation of the effect (the transmission of sound waves).

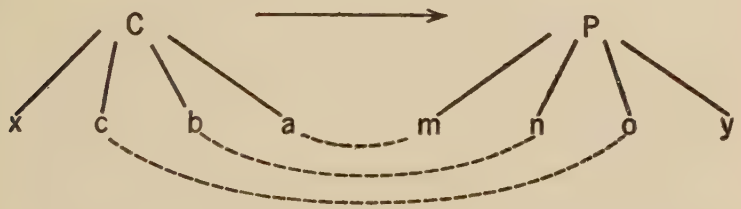
This method is very useful in investigating social phenomena where statistical results can be secured. The comparisons of the graphs representing the results of social surveys will sometimes reveal similarities in curves which suggest causal connections. In such cases the suspected causal relation can then be investigated in individual cases. In cases where the individual phenomena cannot be dealt with as such it may be possible to take the statistical results as given and, by the use of the method of "partial correlation," "isolate" the variables concerned and determine quantitative correlations in keeping with the rules of the method of concomitant variation.

In using the method of concomitant variation we must keep in mind certain principles. In the first place, two things may be varying together in a uniform way (this may be in a *direct* or an *inverse ratio*) and yet the one need not be the cause of the other. As we have previously remarked, the two phenomena may both be the effects of some more remote cause. In the second place, it is not necessarily presupposed by the use of this method that the same ratio between the amount of the cause and the amount of the effect will be preserved indefinitely. For example, there is a correlation between the expansion or contraction of water and the increase or decrease of the amount of heat. This rule holds only within certain limits, for at the boiling point and the freezing point of water we come to critical points where the law breaks down.

In the third place, it is possible that increasing the cause of the phenomenon may give rise to incidental effects which neutralize the main effect. Arsenic is a poison which causes death, when taken in certain amounts.

But if too much is taken this may act as an emetic so that it does not produce its "normal" effect. Many persons overlook this principle when it comes to taking medicine or buying insurance; they suppose that if a little bit is a good thing twice as much doubles the beneficial results. This by no means follows.

The *Method of Residues.*—Mill's statement of this method is as follows: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents." If we analyze a phenomenon, P, with which we are already familiar, into its parts and find that so much of the effect is due to a known amount of the cause, we infer that whatever is left in the effect must be due to something in the cause which is in addition to that part of it which we have already isolated and quantified. This method is applicable wherever we have what we have previously called *resultant* effects, but it is not always applicable in connection with *emergent* effects.[1] In symbolic form this is:



This method was used in connection with the discovery of the planet Neptune. There existed certain unexplained peculiarities in the movement of the planet Uranus; the calculated position and the actually observed position did not coincide. So much of the effect was ex-

[1] In this connection the reader should refer back to our discussion of definition by giving the *genetic formula,* on page 141.

plained as was known to be due to the known causes (the attraction of the other bodies of the solar system in accordance with Newton's law of gravitation); it was inferred that the remainder of the effect (the discrepancy between the two positions) must be due to the presence of some additional cause, in the form of the existence of another and as yet undiscovered planet which was also contributing its increment of influence. The position of this new planet was calculated theoretically and telescopic observations bore out the truth of the calculations. In this way one of the most brilliant illustrations in the history of science of the predictive ability of a quantitative science was presented. But it will be noted that in this case we are dealing with resultant effects, for "vectors" can be added and subtracted. In the biological sciences properties are of a non-additive character even more frequently than in physical science. Here the method of residues is unreliable.

This completes our survey of Mill's methods. Before taking up the more theoretical aspects of causation it will be well for us to summarize the various fallacies which can arise in connection with inductive procedure.

The Fallacies of Inductive Logic

The *Fallacy of Post Hoc Ergo Propter Hoc.*—One common fallacy of inductive reasoning, as we have seen, is that of supposing that if one thing happens after another the thing or event which precedes is therefore the cause of the thing or event which succeeds. This is an error which primitive man was especially addicted to, but the human race has by no means outgrown this tendency. Thus, to illustrate, if a person takes some medicine to cure an illness and then gets well he is very likely to ascribe his recovery to the taking of the medicine.

The *Fallacy of Ignoring Negative Instances.*—As is

readily seen, one can put forth what appears to be imposing evidence for almost any thesis which the human mind can conceive of, *provided one is permitted to select the cases.* By ignoring (either deliberately or unintentionally) the negative cases one can make almost any hypothesis or law appear plausible.

The *Fallacy of Hasty Generalization.*—This fallacy is related to the immediately preceding one. However, one may "jump to conclusions" without ignoring negative instances in every case. In statistics, for example, this may occur when the coincidence of two "central tendencies" of similar curves is taken as proof of a causal relation between the variables concerned in the graph. Francis Bacon pointed out that we must be on our guard against this tendency to anticipate nature.

The *Fallacy of Neglected Aspect.*—This fallacy occurs when for some reason evidence is not forthcoming which we ought to have if we are to arrive at an adequate statement of the facts about a certain situation. Sometimes, from the very nature of the case, this evidence is hard to get. In other cases laziness or prejudice may lead us to overlook relevant evidence. If the proprietor of a pool room generalizes about students and states that all college students are lazy and worthless he is guilty of the fallacy of neglected aspect.

The *Fallacy of False Analogy.*—As we have previously observed, while reasoning by analogy is one of the most fruitful modes of thought, it is also one of the most misleading. The fallacy of *false analogy* occurs when we infer invalidly that because one thing is like another in some respects it resembles the other in additional respects. False analogies may arise as the result of the following errors: (*a*) Basing an analogy on resemblances which are superficial; (*b*) basing an analogy on too few resemblances; (*c*) failing to evaluate the points of sim-

ilarity and difference so as to give due weight to the points of difference no less than the points of similarity.

The following is an example of a false analogy. Every year at Easter time many religious thinkers discuss the problem of immortality. It is a common occurrence for the advocate of the view that the soul survives the body to point out how each spring nature has a new birth, wherein creatures which were dormant during the winter again come back to life in renewed activity. By analogy it is then inferred that man also has a rebirth, in which his soul enters into a new life in a manner no less remarkable than the revivification of the flowers in spring.

It is left to the reader to criticise this analogy and point out the essential points of difference between the "rebirth" of plant life and the supposed "rebirth" of the soul.

Chapter V

THE THEORETICAL FOUNDATIONS OF INDUCTION

WE have said that the goal of science is the establishing of generalizations of the type, *All A is B*. We have insisted that such formulæ, when given concrete interpretations, are the essence of causal laws. A causal law is simply an expression of a constant rule among variable quantities. In general the process of establishing such invariable relationships is a process of forming classes and stating some uniformity of correlation between these classes. This is so because, to repeat, the function of classification is to make possible the assertion of propositions of an inductive type. For instance, the statement that friction causes heat, which is a causal law of an inductive type, means that the class of phenomena included under the term "heat" is included in the class of things correlated with the presence of friction.

The process whereby we establish such correlations follows this general pattern:

$x_1 < y$ (this man is able to think)
$x_2 < y$ (that man is able to think)
$x_3 < y$ (and that man is able to think)

$\therefore$ Some $x < y$ ($\therefore$ Some men are able to think)

If we are willing to make the *inductive leap*—using as our intellectual spring-board the principle of the *uniformity of nature*[1]—we arrive at the formula, *All X is*

[1] The principle of the *uniformity of nature* is stated on page 204; it is explained in detail in the last part of the present chapter.

Y; all men are rational. This formula deserves as much to be called a "law" as any other inductive generalization. And all so-called causal laws are of this form. Many persons will profess not to see any causal relation between man and thought. But in reply it may be argued that there is no less causal connection here than in any admittedly genuine case of causal connection. The statement that the earth attracts physical bodies in accordance with the inverse square law implies no more dynamical connection (in spite of the popular belief to the contrary) than the statement that men are mortal, or that men are rational. The justification for this view will come through a somewhat roundabout way.

The method whereby we attain the formula, *All A is B,* is through the process of *enumeration.* Examining this statement more closely we find that the process of induction is a process of *sampling.* Nature is obviously divided into *kinds.* Gross perception reveals differences between objects. Even dogs, for example, will distinguish between men (as a class) and other dogs. Hence it is clear, that when we inspect anything the classifying tendency is already at work; out of this relation between a knowing organism and the "sensible muchness" of the environment (as William James called it) order begins to emerge. We take these gross classes and begin a more detailed analysis of the individuals. Here the sampling process enters. Let us suppose it is potatoes we are examining, and that we are interested in testing whether all these things which look so much alike in most respects are also alike in this particular respect, that each and every one of them possesses "eyes." If we have a small heap, or if in general our group is of a finite number, we can examine every one. But if we have an indefinite number of individuals we may make a "random selection." If our samples are "fair samples" of the

class as a whole, and each of them possesses eyes, we are inclined to jump to the conclusion that all potatoes have eyes. We therefore establish it as an essential characteristic of the class potatoes that they have eyes.

If at some time in the future we should come across individuals failing to possess eyes, in accordance with our law, we then face the alternative of either (*a*) reducing a universal proposition to a particular (i.e., "all" to "some") or (*b*) of making a new class and excluding this new class of eyeless creatures from our previous class, thus saving our generalization or inductive law.

Ultimately, therefore, all causal laws, as well as all generalizations not ordinarily considered to involve causality, are based on enumeration. History, or the record of what is, is the basis of statements of causality. In the last analysis we can only say that observation reveals that this is that, *A is B*. All cases of causality are cases of summary description. An illustration will clarify this. Observation and enumeration have brought us to the conclusion that all planets revolve around the sun in elliptical orbits. Now we ask—following popular belief in "explanation"—*why* is this the case? What is the *cause* of the movements of the planets of the solar system in accordance with Kepler's laws of planetary motion?

In reply to this question we may be offered Newton's law of gravitation. We may be told about how the circle (or more strictly, the ellipse) is a compromise between the centripetal force of the attraction of gravitation, tending to draw the planet into the sun, and the centrifugal force tending to throw the planet out into space on a tangent. But then, we ask, What is the explanation of the attraction of gravitation? What is the cause of centripetal force? Here we come to the end of our rope, as we always do if we push an inquiry far enough. Explanation, like definition and classification, eventually re-

fers back to the brute facts. You point to the thing in question, label it with a term, but you can go no further. *Gravity* is a simple case of enumeration, of summary description.

While the citing of authorities does not prove a point, it lends weight to a viewpoint. Accordingly the following quotations are set down. As far back as 1889 Professor J. H. Poynting said (in his *Address* to the British Association): "We must confess that physical laws have greatly fallen in dignity." Professor Poynting goes on to say that a law of nature explains nothing (in the sense that it tells us *why* things happen); it has no governing power; it is but a descriptive formula which careless people have sometimes personified. This same view is taken by Karl Pearson in his *Grammar of Science* (edition of 1900, p. 99), where he says: "The law of gravitation is a brief description of *how* every particle of matter in the universe is altering its motion with reference to every other particle. It does not tell us *why* particles thus move; it does not tell us *why* the earth describes a certain curve about the sun. It simply resumes, in a few brief words, the relationship observed between a vast range of phenomena. It economizes thought by stating in conceptual shorthand that routine of perceptions which forms for us the universe of gravitating matter." So far as we can now see, it does not appear that the more recent work of Einstein in this field negatives this viewpoint concerning the nature of the law of gravitation.

It must not be overlooked that this confession of theoretical ignorance does not detract one iota from the practical value of the notion of cause. *A cause is that by getting rid of which we can get rid of the effect,* or *vice versa.* Hence Mill's methods are still as useful as ever. One can get rid of the cause of a headache without knowing why the cause produced the result, or why the

removal of the cause is followed by the disappearance of the effect.

The Meaning of Causation

Thus far we have confined ourselves largely to a discussion of the meaning of the term, "scientific law." Now we must turn our attention to the implications of our view for causation.

Anthropomorphism.—Primitive man, as we have seen, argued by analogy. As a result he came to consider the phenomena of the external world as being produced by the same type of cause that he was familiar with in his own inner experience. Since human conduct is purposive, being the expressions of the effort to realize or achieve the goals or ends set up, primitive man argued that the phenomena of the external world are likewise to be understood as the result of strivings on the part of personal agencies or vital forces to accomplish certain end results. Thus the *teleological* or purposive type of explanation was started on its long and active career.

Another fallacy contributed to the growth of this type of explanation. As we have seen, man has the power to deal with abstractions. He can label these products of abstract thought or conception with terms. It is a natural but unfortunate tendency to suppose that whenever we have a word or term there must be some reality (entity) that corresponds to that term. In this way man has come to personify and reify what we now regard as functions or processes. Forms of behavior are treated as things. And so it has come about that the term *force,* which is a name designating an important type of process (i.e., a change of spatial relationship), has been *hypostatized* (or "thingized") into an entity. This projection of our sense of effort or muscle strain into the external world to cover the processes of nature, and this

reification of force as a cause which produces the movements of matter, gives us what H. Poincaré once termed "anthropomorphic mechanics." It is but one phase of the animistic system of thought devised by the mind of primitive man.

This type of explanation has by no means been completely "devitalized." It is still with us in popular thought. Not only was classical physics "haunted," but, as one writer has said, anyone today who still believes in "causality" is superstitious. The common sense notion still prevalent, that the cause must necessarily precede the effect, is a lingering survival of this earlier view. This follows from the notion of causation as purposive realization of ends, for purposes always precede the course of action whereby the goals of conduct are realized.

Causation in Modern Science.—Modern science has rejected this animistic conception of causation; this is true at least for the sciences concerned with the external world. What will happen to causation when human conduct is up for explanation we cannot yet determine. But certainly at present the tendency to think of mind as the causal agent which produces human response is on the decline.

The reasons for the depersonalization of causation in the physical science are several. The same things which have helped change our notions about the nature of natural law have also helped to change our notions about causation. The first great sceptic who voiced his suspicions of the traditional interpretation of causal connection was David Hume.

Hume's destructive criticisms of induction were based on the denial that we have any right to assert that there is any *necessary* connection in nature corresponding to the connection of the ideas of cause and effect in our own

mind. According to Hume, we observe two things happening together a number of times. If these occur together constantly we come to associate them by the appropriate law of the association of ideas. Thus, for example, we come to believe that friction causes the match to ignite. By an act of faith in the uniformity of nature we assert that there is therefore a necessary connection between the cause and the effect, so that if in the future the same cause is present the same effect is produced.

Our faith in this principle that nature must behave in similar ways under similar conditions is so strong that if we come across a case where we have a cause which is not accompanied by what experience has led us to believe is its effect we say that the two situations are not exactly the same, however analogous they appear superficially. Or we assert the existence of some unknown counteracting cause which nullifies the original cause.

Modern science has followed Hume in rejecting the notion of necessary connection. Science has striven more and more to purge physics of its subjective and anthropomorphic elements. Dynamics, for instance, has become less and less "dynamical." The only case where there is any uniform transference of "energy" from the cause to the effect is in elementary physical processes. But even here critics are asserting that "energy" is only a name for a type of behavior, and that we must not personify it into an entity. The real thing in nature is a fairly reliable connection between sets of phenomena. Thus the law in nature of the *conservation of energy* (which has been the chief scientific justification for the principle of the uniformity of nature) is only a *habit of nature;* and nature, like a human being, may occasionally break old habits and form new ones. The law merely summarizes a stream of phenomenal impressions, as

Hume would say. But for that matter, we get the same type of uniformity of connection in a moving picture, and this, as we have seen, is an illusory connection. What really carries over from one event to another is not energy, or a necessary physical connection, but a uniform mathematical relationship. And we have yet to establish the right to identify mathematical constancy with physical realities.

This conclusion now gains added support from the implications of relativity physics. The notion that a cause is a force and that forces are causes is undermined by Einstein when he treats the movements of gravitating bodies as due to the "curvature" of space. The relativity of motion (and hence of the *acceleration* which was supposed to be the test of the presence of a "force") leads to the relativity of all "forces." Moreover, the relativity of time measurements undermines the notion of absolute precedence and succession in a temporal series; so that, as Professor G. N. Lewis[2] remarks, relativity geometry "removes the last foothold in the physical sciences of the concept of temporal causality." But this is a disputable point and we must avoid the appearance of dogmatism in the statements we make.

Modern science thus appears to lend support to those conclusions which some philosophers, mainly those of a positivistic[3] bent, had already arrived at. Such complications in the determination of causal connections as are referred to by the terms the "plurality of causes" and the

[2] *The Anatomy of Science,* p. 134.

[3] *Positivism* is the name which the French philosopher, August Comte (1798-1857), gave to his point of view. According to Comte human thought, in its evolution, passes through three stages: the *religious,* the *metaphysical,* and the *positivistic,* or *scientific.* Primitive man explains the events of nature by supposing that the gods, or personal spirits, cause these events. In the metaphysical stage of explanation trans-empirical principles (such as the "vital force" of biology) are employed to explain the behavior of things. In the last stage, the scientific, all anthropomorphic elements are discarded. Science does not attempt to explain *why*

"infinite regress of causes" have been names for difficulties which threw doubt upon the scheme of nature according to which "cause" and "effect" were held to be realities possessing unequivocal objective sanctions. This view is now open to serious doubts. Whether the notion of causality will ever recover its earlier realistic implications is difficult to say.

To sum up our argument thus far we may point out that at present the notion of causation is held by most scientists to be synonymous with the mathematical concept of functional relationship. Causation is held to be a case of what is termed *dependent variability.* If $y = f(x)$, so that every time you assign a value to x you thereby determine a corresponding value for y, then you have as good a case of causation as you can ask for. The relation between such variables can be stated graphically by the use of coördinate paper. As a result of a series of experiments you can plot the equation of the curve obtained in terms of Cartesian coördinates (the x, y, and z axes, though in graphs only the first two are ordinarily used). In doing this you are fulfilling the function of science, which is to draw curves representing the functional dependence of the different variables of the experiment. Thus mathematics and natural science go hand in hand. This common basis consists in the use of formulæ. As already pointed out, the essence of a mathematical formula and a scientific law is that they are ex-

things happen, in the sense that it gives the *purpose,* but is content merely to *describe how* they happen.

This view is frequently linked with the similar idea known as *phenomenalism,* a doctrine associated with the name of David Hume (1711-1776). According to this doctrine, the mind can know only phenomena; science is therefore restricted to the realm of sense perception. Man can know nothing of what Kant later called "things-in-themselves," or *noumena.*

Combining these two doctrines, positivism and phenomenalism, we come out with a theory similar to that advocated by Karl Pearson in his *Grammar of Science.*

pressions of a *constant* rule among *variable* quantities. An illustration from mathematics and from natural science will clinch the point.

Suppose that we have a rectangular solid with the volume V, where a and b are respectively the length and the width of the base, and c the altitude of the solid. In this case $V = f(a, b, c)$. This functionality is more specifically expressed as $V = f(a \times b \times c)$. Now let us take a parallel case from an empirical science. In meteorology a concrete example of the same type of functional dependence would be as follows: the state of the weather, S, depends upon the temperature, T, the humidity, H, the direction and velocity of the wind, D and V. Then $S = f(T, H, D, V)$. In either case we may say that the formula gives us a causal law. The cause of the change in the volume of a solid is a variation of the length of the rectangular coördinates, and the cause of the state of the weather is a variation of the meteorological variables. This, it will be observed, takes care of the pragmatic element in causation, which we have also emphasized. For, as your purposes dictate, you can select the appropriate variable which your graph shows you is related to what you please to call your effect and vary it in whatever manner you please in order to modify, produce, or eliminate your effect. This point of view has the advantage that it is independent of whatever attitude science may take in the future towards the notion of "necessary" connection and the problem of temporal sequence in causal relations.

Induction and the Uniformity of Nature

No one doubts the practical necessity for a faith in the uniformity of nature. One illustration of the need for such a principle in science is illustrated by Clerk Maxwell in his *Discourse on Molecules* in 1873, when he said:

"In the heavens we discover by their light, and by their light alone, stars so distant from each other that no material thing can ever have passed from one to another; and yet this light, which is to us the sole evidence of the existence of these distant worlds, tells us also that each of them is built up of molecules of the same kind as those we find on earth. A molecule of hydrogen, for example, whether in Sirius or in Arcturus, executes its vibrations in precisely the same way."

And yet we do not know whether the unquestioned usefulness of the principle of the uniformity of nature is grounded in the structure of the external world or whether it has its origin in the psychology of the human mind. But it is this precise rift between theory and practise which makes it imperative to investigate further this foundational problem.

At the present time a number of logicians are trying to find answers to Hume. The chaotic state of the situations is indicated by Dr. A. N. Whitehead[4] when he informs us that "the problem of Induction is the despair of philosophy—and yet all our actions are based on it." Bertrand Russell[5] takes the same view when he asserts that "Induction raises perhaps the most difficult problems in the whole theory of knowledge. Every scientific law is established by its means, and yet it is difficult to see why we believe it to be a valid process."

This problem of finding a rational ground for induction has called forth so much attention that in describing the situation we might almost say that contemporary philosophers and logicians have developed an "induction complex." Now it is a common-place of methodology that before we attempt to solve a problem we must be sure that the problem is a genuine one. Perhaps it would

[4] *Science and the Modern World*, p. 35.
[5] *Philosophy*, p. 14.

be well therefore to raise the previous question as to whether we have here an illustration of a meaningless (or at least insoluble) problem. Accordingly, let us examine this belief and see whether in fact there is some unusually profound mystery about induction and the uniformity of nature.

One reason which is sometimes given for the view that the principle of the uniformity of nature cannot be used as the foundation for the erection of causal laws is that this principle is itself a result of induction. The only reason we have for assuming it to be true is that in many cases experience has shown it to be true. But, the reply is suggested, you cannot take as rationally self-evident or certain a principle which has only probable validity. Unless you can give some good reason for it, you cannot assert that nature will be uniform in behavior in the future simply because she has been more or less so in the past. As a matter of fact, nature cannot behave in the same way in the future as she has in the past because nature never exactly repeats herself. There are no absolute uniformities, and the statement presupposes ideal conditions which are never realized in practise. In other words, the principle, to be true, must carry us beyond experience, but empirical science, as the study of the relationships of experimentally observed facts, has no right to go beyond experience, the actual.

The situation here is somewhat analogous to the status of the "law" of probability. Professor G. N. Lewis once pointed out to the writer that physicists have assumed that the mathematicians have proved the law of probability, while the mathematicians have assumed that the physicists have proved it. In reality neither has proved it. We are in the same situation when scientists assume that there must be some philosophical justification for

the principle of the uniformity of nature and philosophers assume that science has justified it.

This way of stating the matter at once brings us to a general type of situation with which we know how to deal. Where such situations occur the only way to avoid circularity is to rest the case (it may be to justify an axiom, a definition, or a principle) on postulation, or self-evidence, or faith, or intuition, or the pragmatic usefulness of the concept, or proposition. In mathematics the justification for an assumption is intuition or postulation. In empirical science the justification for an assumption is its pragmatic usefulness. *But no matter what the grounds for our choice may be, there can be no further reasonable grounds for the choice, and no theoretical justification for the principle, in the sense that its necessity can be deduced from something else.*

And now where does this leave us with respect to the problem of induction? For one thing, it places our faith in the uniformity of nature in that class of concepts and propositions which are *primitive,* in the sense that they cannot be derived from anything which is logically prior. But there are many such concepts and propositions, which therefore suggests that induction has no unique status as a special problem. It is doubtful whether the problem of induction is any more intricate or mystifying than any other phase of the so-called problem of knowledge.

Does this undermine the utility of the doctrine? Not at all! No matter what happens to the theory of the uniformity of nature, in actual practise we will always be forced to employ something which amounts to the same thing. And behind such a principle we will never be able to go in theory.

If this is the correct viewpoint the opposite belief that at some future time thinkers will be able to discover or

invent some rational justification for the success of the inductive sciences is mistaken. Perhaps it is Mr. Russell's occasional excessive rationalism which leads him to the belief that induction is perhaps the most difficult problem in the whole theory of knowledge. This whole problem may vanish if we are but permitted to restate a proposition which, as we saw in our opening chapter, Mr. Russell sets down as fundamental to his type of scepticism. In his *Sceptical Essays* (Ch. I) he tells us: "It is undesirable to believe a proposition when there is no ground whatever for supposing it to be true." Now, we ask, Is it not possible that Mr. Russell and those who accept this viewpoint create for themselves this problem of induction by their excessive logical parsimoniousness?

If it is true that we should not believe any proposition until we have good ("positive") grounds for believing it then of course as philosophers and scientists we should not believe in the uniformity of nature until we have justified that belief. From this point of view the burden of proof for accepting the principle rests on the man who believes in it and employs it. But is this mode of procedure true to the human mind as we observe it to work generally? The evidence suggests an answer in the negative. And if the minds of philosophers work like the minds of normal (average) people, then the burden of the proof ought to rest on Mr. Russell to show why we should not believe any given proposition.

A study of the psychology of suggestion and hypnosis will demonstrate that we actually tend to believe as much as we can, and that the only reason we have for being sceptical about the truth of any proposition is that it conflicts with something else which we already believe. In accordance with that doctrine Russell's proposition ought to be restated as follows: "It is undesirable to reject a proposition when there is no ground for believing it to

be false." That this statement comes closer to expressing the truth about the testing of our beliefs may be shown by an appeal to the following case.

Let us suppose that a teacher proposes to one of the members of the class that she come before the group and sing for them. In all probability the girl would refuse. She would immediately think of every reason why she should not do this: she is not dressed for the occasion; she does not know how to sing (which sometimes is sufficient reason for not singing); and so on. But if the teacher can dispose of each objection as it arises, the young lady, not being able to think of any more reasons why she shouldn't, is eventually forced to sing for the class. A hypnotist would be able to get the young lady to do the singing where the teacher would probably fail because he knows how to get the young lady into the state where he can limit her field of consciousness so as to exclude the influence of "collateral" ideas, which would otherwise tend to inhibit what normally seems like an absurd action.

Probably the physiological basis for this narrowing of the field of consciousness is explained for us in the doctrine of the irradiation of nervous energy. Normally stimuli which get into the cortex tend to irradiate over the functionally associated neurones, but if this energy can be drained into one channel so that there is little irradiation, then there is no association of ideas, none of the criticism which constitutes intellectual vigilance, and the stimulus finds its way out to the muscles in the form of a response. Thus we become slaves to ideas, compelled to follow out the actions which they dictate. It must be admitted that the physiological details underlying this process of the limitation of the field of ideas are still obscure, but the general notion that the difference between states of sleeping, hypnosis, dreaming, and waking

is due to the degree of irradiation of the cortical inhibitory process is in line with the work of the psychoanalysts and the physiological work of Pavlov.

If this is a true analysis of the belief-content of our minds, it becomes clear that we tend to believe as much as we can. It may, of course, be argued that philosophers should not take their standards from the average man. But in reply it can be pointed out that the species *homo boobiens* and *homo sapiens* (including philosophers) are both members of the genus humanity. The minds of the majority of human beings very probably work as we have outlined. The laws of logic and the principles of inference have been established by the social test working over a long period of time.

This view by no means encourages a recrudescence of the religious and political fanaticism which it is the purpose of Mr. Russell's type of scepticism to prevent. It means that the more one knows the more critical he becomes of incoming ideas. If a belief is inconsistent with our previously accepted body of knowledge, if, like the belief in supernaturalism, it threatens to destroy intellectual integration, we are then faced with the alternatives of rejecting the new idea or modifying our old beliefs so as to make them consistent with the new idea.

In reply to the question, When can an idea be accepted as true? Mr. Russell would probably say: When we have positive grounds for believing it to be true. But according to the present view we would say: When you can't find anything wrong with it. In either case, however, an idea is held to be false when it fails to check up with the other ideas which are accepted as true. But the positive grounds for truth turn out to be the absence of conflict with other ideas accepted as true. While in some cases the results which follow from Mr. Russell's principle and from the present view would be about the same, in

the problem of induction the question of whether you do or do not have a problem is settled by the choice you make between the alternatives which have been suggested.

By way of illustrating our general viewpoint let us appeal to an analogy. If all chemical elements exhibited the property of magnetic susceptibility the physicist would not regard this property as standing in need of explanation. If someone were to ask him why elements exhibit the property of magnetism he would very properly reply with another philosophical question, Why shouldn't they? But in actual fact only a few elements (notably iron) exhibit this property. Hence the physicist considers it a legitimate problem to investigate why iron exhibits magnetic properties. Now to return to our own problem. If we should discover a species of creature who did not believe in the uniformity of nature and for whom induction is an invalid process there would be grounds for a philosophical investigation of the difference in the make-up of men and these creatures which would explain this difference in point of view. It would also be a legitimate problem to investigate which of these two races of creatures is correct in its belief. But aside from a few philosophers we have no evidence of the existence of such a race of creatures. An "animal faith" in the uniformity of nature seems to be in accord with the not-too-sophisticated minds of humanity in general. Why then should philosophers suppose that they need to solve problems which in the last analysis turn out to be insoluble?

Chapter VI

MEASUREMENT, STATISTICS, AND PROBABILITY

OBSERVATION in its cruder and more primitive forms consists in noting qualitative distinctions in the external world mediated by sense perception. These are then grouped into classes. Relationships are on this level confined to products of comparisons between gross classes. By this time men have learned to count these classes. But for the most part their knowledge is still of a qualitative sort. Men can say that one thing is bigger, or heavier, or older than another. Thus the foundations are laid for quantitative categories of a more refined sort which come later. The technical units of measurement of masses, lengths, times, etc., which we now possess, are products of a more sophisticated stage of human thought.

As time goes on better units are devised, units which serve a wider variety of purposes. Also it becomes possible in time to measure finer differences, differences not only *between* classes but differences *within* classes. That is to say, first of all men note the difference, e.g., between red and yellow objects, and then later they learn to measure differences within red and yellow colors.

The dependence of science on measurement has frequently been pointed out. It is this need for securing quantitative knowledge which seems to make science such a prosaic thing. As Lord Kelvin has pointed out:

> Accurate and minute measurement seems to the non-scientific imagination a less lofty and dignified work than looking for some-

thing new. But nearly all the grandest discoveries of science have been but the rewards of accurate measurement and patient, long-continued labor in the minute sifting of numerical results.

The most significant phase of the evolution of each science is quantification. We have already touched upon some of the reasons why mathematics should be the medium through which the results of scientific research are summarized. The following quotation from Professor Whitehead[1] gives us an additional reason for the fruitfulness of the marriage of science and mathematics:

> It is easy, therefore, to understand that we want to describe the connection . . . between external things in some way which does not depend on any particular sensation, nor even on all the sensations of any particular person. The laws satisfied by the course of events in the world of external things are to be described, if possible, in a neutral universal fashion, the same for blind men as for deaf men, and the same for beings with faculties beyond our ken as normal human beings. . . .
>
> Thus it comes about that, step by step, and not realizing the full meaning of the process, mankind has been led to search for a mathematical description of the properties of the universe, because in this way only can a general idea of the course of events be formed, freed from reference to particular persons or to particular types of sensation.

The reason why mathematics adapts itself so nicely to this demand of science is readily shown. The essence of science, we have reiterated, is uniform correlations between classes of facts. Science, therefore, does not insist that it have a direct knowledge of any type of reality, such as "matter." Science is concerned with measuring magnitudes and changes of relations of the entities to which those magnitudes are supposed to refer. But the *thing* as such it leaves untouched philosophically. If you want to identify the measure-quantities with the

[1] *An Introduction to Mathematics,* pp. 12-13.

things themselves, well and good—though philosophically this is a dubious act. This view may appear somewhat unusual. It is quite recent as a doctrine of science, though in the form of the doctrine called phenomenalism in philosophy the viewpoint has respectable antecedents.

Probability as the Guide to Life

Time has been said to be the direction in which we are all going without effort and without exemption. From this point of view it is clear that we cannot control time; all we can do is make the best use of it. Unfortunately (or it may be fortunately) consciousness is so limited that we can be aware only of the present and recall the past through memory. We cannot know the future, which is hidden behind an impenetrable veil. But the necessity for making decisions about matters which do not come within the purview of the immediate present is inescapable. We must act with reference to coming events, even though at best they merely cast their shadows before. Accordingly there has developed in human beings an instinctive tendency to anticipate the future through a reliance on a doctrine of chance. For this reason probability has become the guide to life. From the point of view of the logic of chance such ancient schemes of divination as the use of cards, astrology, palmistry, and the like, are only dubious uses of the calculus of probabilities.

There are no contingencies in life which arise concerning which we have more than probable evidence on which to make our decisions. So far as the certainties of life are concerned, Calvinism as a doctrine of predetermination has no sanction either in theoretical logic or in human practise. God may foreknow the future, but men are not gods—as yet.

The only place we get certainty is in pure logic and in mathematics. If I ask, What is the probability that a

given square is a triangle? the answer is that the probability is zero. This type of certainty is a result of definition. In the same way, the probability that Friday will fall on Monday is zero; it is absolutely impossible. But in any inductive science your propositions can only approximate certainty. In the world of nature, laws, beliefs, hypotheses, etc., at their very best can only achieve a high degree of probability. As Poincaré [2] has pointed out, all the sciences are unconscious applications of the calculus of probabilities, so that if this calculus be abandoned the whole of science must be condemned.

This is the newer view. But it has not always been thus. In the past the sciences have been more dogmatic in laying claim to certainty. And it is because of this change in attitude that the theory of probability is taking on more and more importance as time goes on.

In most logic texts the presentation of the subject of probability is confined to the reproduction of the theorems of probability as applied in the calculation of independent and dependent events. The present chapter is written in the belief that the place to become sufficiently well acquainted with the rules to use them is in a course in statistics. The thing which needs to be discussed in logic texts is the *theory* of the procedure, something which is seldom dealt with in treatises on statistics and probability.

It is also a legitimate question to ask what the sense of applying rules may be if you are not sure that the rules are correct. That there is still room for doubting the sacredness of the rules may be illustrated by the following considerations. Let us begin the conventional definition of probability: the probability of an event is said to be the ratio of the number of cases favorable to the event to the total number of possible cases, favorable and unfavorable. Probability, in other words, is represented

[2] *Science and Hypothesis*, 1905, p. 186.

by the fraction $\frac{p}{p+p'}$, where "p" is the number of favorable cases and "p'" is the number of unfavorable cases. Now let us apply the rules (more strictly, the definition). If I ask, What is the probability of Christmas coming on Monday this year? the probability is $\frac{1}{1+6}$, or $\frac{1}{7}$. But, as is readily perceived, this calculation is acceptable only if there is no reason to believe that there is some factor tending to make it fall on one day rather than on some other. Using the example of the throwing of dice, the probability that we will get a six on any given throw of a single die is $\frac{1}{1+5}$. The probability that if we throw two dice both will come up a six is the product of the separate probabilities; that is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. But if the dice are "loaded" the probability is radically altered, so that the laws of "chance" are no longer followed. And so it has been found necessary to add to the above definition (or rule) the words ". . . *provided the cases are equally probable.*"

And so we are compelled to define the probable in terms of the probable. But how can we know when two events are "equally probable"? As Poincaré points out (*Op. cit.*, p. 186), if we insert at the beginning of every problem an explicit convention which tells us when events are equally probable we then have nothing to do but apply the rules of arithmetic and algebra, complete our calculations, and then our results cannot be called into question. But, as Poincaré says, if you wish to make the slightest application of this result you must prove that the convention is legitimate, and you find yourself

in the presence of the very difficulty which was to be avoided. In trying to get around this difficulty some writers put the notion of "equally likely" in the class of concepts which are intuitive and therefore indefinable. Another possibility is to justify the definition in terms of its pragmatic usefulness. But this is no longer a theoretical justification.

The situation here is somewhat similar to that which we have noted in connection with "consistency." In connection with the theory of probability there are two schools of thought: one school of thinkers, called the "cogent reasonists," say that two events are equally likely if there is a cogent reason for thinking them so. The "insufficient reasonists," on the other hand, say that two events are equally likely when there is no reason to think them otherwise. If we are to be consistent with our position as previously enunciated in connection with belief, our attitude will be that of the "insufficient reasonists,"—two events will be held to be equally probable if there is the absence of any positive reason for thinking them otherwise. In that case the burden of the proof rests upon the man who asserts that the dice are "loaded."

Causality and Probability

Previously we have argued that there is a close connection between induction and classification. This is obvious when we are dealing with "perfect" inductions. Thus if we have thirty members in a class and it is discovered that every individual of the class is less than six feet tall, we may state without fear of contradiction that all the members of the class are less than six feet tall. But in many other cases we are not dealing with such perfect inductions. If we have a causal law of the type, "Lightning causes thunder," the total number of cases of lightning has never been enumerated. And so, if on some

occasion when we perceive a flash of lightning, we say, "Now we will hear a clap of thunder," we cannot be entirely sure that this prediction will be verified. We are here moving in the realm of imperfect induction. Since we have made the inductive leap on the basis of a faith in the uniformity of nature, it may turn out that in this case the thunder will not follow the lightning. And the same can be said for any other causal law of an inductive type.

In such cases, where all the instances of the phenomenon have been favorable, there is a simple rule for calculating the probability that in the next case the same cause will be followed by the same effect. If an event, A, has been followed by the effect, B, the probability that in the next case the cause, A, will be followed by the result, B, is expressed by the fraction $\frac{n+1}{n+2}$, where n is the number of times A has happened. Here the numerator represents again the total number of favorable cases and the denominator represents the number of times it has happened plus the possibility of the repetition or non-repetition of the event. To illustrate, if it has been historically established that winter has followed summer for 5,000 years the probability that winter will follow summer this next year is $\frac{5,001}{5,002}$. As the number of times this sequence recurs becomes larger the probability that, given another summer, it will be followed by winter becomes larger. The probable connection between the given cause and its appropriate effect approaches unity as a limit. But this limit, which is absolute *certainty,* is never reached in this world, for to reach the limit would require an infinite number of cases, and this cannot be obtained by counting the finite number of cases which inductive science is restricted to. Hence inductive generalizations never give us more than very high probability.

The problem we have just discussed is this: Given a known cause, What is the probability that it will be followed by its usual effect? The inverse problem is that of reasoning from known events to probable causes. This is much more complicated. Among other things the "plurality of causes" enters in. Because of the fact that a given effect can sometimes be produced by a variety of causes (the causes of headaches or deaths are numerous), the probability that a particular effect was due to a certain specified cause is less than the probability that a particular cause will be followed by a specified effect. Thus here again we see that there is an intimate connection between the establishing of causal connections and the calculus of probabilities.

The probability attaching to some statement about an individual is the probability of the truth of the proposition when applied to the class to which the individual belongs. The probability that any individual, A, is mortal depends upon the probable truth of the statement that all men are mortal. We cannot regard this proposition as absolutely certain. It is probably as certain as anything else in this world, for the number of men who have already died is considerable. But in any case, in order that the truth of any inductive generalization can be highly probable there must be a *constancy of behavior of the class as a whole.*

It is this remarkable constancy of behavior of classes or statistical regularity of groups which makes the reliable functioning of insurance companies possible. This regularity of statistical ensembles is sometimes personified into a law. Buckle, for instance, assumes this interpretation in the following statement from his *History of Civilization* (Vol. I, p. 25):

> In a given state of society a certain number of persons . . . must put an end to their own life. This is the general law, and

the special question as to who shall commit the crime depends, of course, upon special laws, which, however, in their total action, must obey the large social law to which they are all subordinate. And the power of the larger law is so irresistible that neither the love of life, nor the fear of another world, can avail anything towards even checking its operation.

It is this association between the constancy of classes and probability which forms the bridge between induction and probability. This uniformity of class behavior is frequently established by repeated observation, and the result represents a kind of statistical constancy of high stability. This again demonstrates how fundamental probability is in science, for the validity of statistical investigation rests upon the validity of the theoretical foundations of the calculus of probabilities.

Probability and Statistics

Measurement, the use of statistics, and calculations of probability have in common the process of enumeration. But before you can count there must be distinct units to which you apply the number system. These elementary reflections suggest some fundamental principles which are presupposed in all types of calculations. But let us first glance at some of the assumptions involved in the theory of probability:

1. There must be individuals to be enumerated.
2. The individuals must be sufficiently similar to be numbered as members of classes.
3. These individuals must engage in the type of behavior called events.

Let us discuss these in the order given.

The first postulate, that there must be individuals to be enumerated, is a postulate of discreteness. The number series used in measurement is a system of discrete units,

so that in counting a one-to-one correspondence between the numbers and the things to which the numbers refer can be established.

In his *Treatise on Probability* J. M. Keynes, who has there given us the most thorough discussion of probability from a logical point of view, calls this postulate the *principle of the limitation of variety*. This means that in certain aspects nature is not infinitely variable, but is built up of a limited number of constituents. In the words of Keynes, "the amount of variety in the universe is limited in such a way that there is no object so complex that its qualities fall into an infinite number of independent groups" (p. 258). Thus, for instance, if we are going to talk about "ivory, apes and peacocks," we must assume that anything we may say in the future about any of these possesses a probability of being true because, e.g., the new ape, through its analogy with the old in the various denumerable qualities, can have the proposition established to hold for the old members of the class distributed over it also. If the qualities and behaviors of apes were infinite in number no statement of a specific nature about apes could be made.

The second postulate is a postulate of identity of individuals of a class. The only place where we get absolute identity is in mathematics. As previously intimated, it is doubtful whether each electron is exactly like every other, but if these differences fall outside the limits of the most refined experiments they are negligible. For practical purposes the units of classes about which we generalize are sufficiently alike *with respect to the qualities which are the basis of the classification* that any other differences may be ignored.

This postulate is called by Keynes the *principle of atomic uniformity*. He tells us that many, if not all,

processes can be considered as compounded of small changes according to simple mathematical laws. In his own words, "The system of the natural universe must consist, if this kind of assumption is warranted, of bodies which we may term (without any implications as to their size being conveyed thereby) *legal atoms,* such that each of them exercises its own separate, independent, and invariable effect, a change in the total state being compounded of a number of separate portions of the preceding state" (p. 249).

Our third postulate tells us these atomic units must engage in the type of phenomenon called *events.* This must be so if probability is to be defined in terms of a *ratio between events.* As used in calculations of probability, the term event refers to something individual and atomic. More recently, in relativity theory, an event has been defined as the intersection of the "world-lines" of two particles.

Thus far our comments have been confined largely to the subject of probability. Let us next glance at the way in which these ideas reflect themselves in the field of statistics. But first let us retrace our steps a bit.

In a previous chapter the point was made that in experiment we are concerned (*a*) with the measurement of the variables correlated with the phenomenon which we term the effect. But this is by no means the limit of our investigation. We frequently find it desirable (*b*) to have a measure of the variation of the variable we have isolated. And in other cases (*c*) we also want to know the relation between several variables which are varying together. We noted in discussing the method of concomitant variation that it is sometimes possible to determine causal relations of the type (*c*) if we can find some systematic correlation between the variables. It is here that statistics is of use.

The aim of statistics is to deal with variables affected by a number of causes. In such cases the causes will either be unknown, or of such a complexity that even though we know qualitatively the types of causes involved we have no quantitative values telling us how much of each of these causes is effective. Here the use of statistics enables us to form probable judgments as to what will happen. The constancy of class behavior is here a statistical constancy, but because of our ignorance of the finer details of such uniformities we cannot be certain that this constancy will be maintained in the future.

In the use of statistics the following types of measurement are useful:

1. An average value which will represent the value of the variable.
2. A measure of the variability of the items with respect to the average of value chosen.
3. A measure of the extent of the relationship between two variables which are statistically associated.

Even in measurements which are not ordinarily regarded as of a statistical nature the first type of value is employed. If you are testing any physical law, such as the law of falling bodies, you perform a number of experiments and then take the average of the results. In a series of observations any given measurement is likely to be "off" to one side or the other to a greater or less extent, though generally the results are scattered about a mean position in such a way that the number of deviations from the mean position is inversely proportional to the amount of deviation. If the figures of the series represent the separate measurements of a variable any deviations from the average can be regarded as "errors." The character and the size of these errors will depend on the nature of the instrument used and the ability of the observer to make experimental observations. In experimental investigations there is always a "probable error"

and there may be a "systematic error" due to conditions peculiar to the experiment.

All these errors, if known to occur, can be allowed for. The way to correct for probable error is to take the results of the series of experiments and choose the limits within which it is as likely as not that the truth will fall. It is always easy to calculate the arithmetic average. As everyone knows, this is the sum of the numbers divided by the number of figures itself. There are other ways of calculating these various kinds of errors, and the type of calculation employed will depend in part upon the nature of the problem and the variability of the results. In most empirical measurements the probable error is obtained through the use of the method of "least squares," but this is too technical for treatment here.

Thus it appears that there is always something arbitrary (or something anthropomorphic) in all our calculations. Some social scientists who make a fetish of "objectivity" seem to think that statistics is a kind of magic formula which excludes all subjectivity from their science; but this view is not entirely correct. The personal equation, kicked out of one door of the domain of science, returns through another, and appears in the statistical values which are chosen. As one witty critic pointed out, after having calculated the probability of an error, it is then necessary to calculate the probability of an error in the calculation. Thus from the point of view of theory we are launched upon an endless regress of calculating the correctness of the just preceding calculation. But practical considerations make it necessary to "take a chance," so in science we agree upon a convention as to what kind of a chance we are to take; and since all of us take the same kind of chance, no one knows the difference.

The Dangers in the Use of Statistics

Everyone seems to be aware that "you can prove anything by the use of statistics." And yet individuals are frequently making extravagant claims as a result of a statistical investigation of one sort or another. This must be due to the natural human tendency for a man to fall in love with his own theories.

When it comes to asserting causal relations on the basis of positive correlations between the variables of a statistical survey special care must be exercised. In the first place, if there is any *interpolation* or *extrapolation* this is a source of danger. In the second place, even though the curve which states the relations between two variables contains no such uncertainties, we find that the same equation for the curve which represents our empirical results can in many cases be applied to disparate and unrelated phenomena. The mere fact that you can see a similarity between two smoothed curves which represent the result of statistical investigations is no certain grounds for asserting a causal relation. For instance, one of the few laws of psychology which we have is the Weber-Fechner law of psychophysics. This law tells us that as the intensity of the external stimulus increases in a geometric progression the corresponding sensation increases in an arithmetic progression. This law was established as a result of repeated experiment and is really a statistical generalization dressed up as a causal law. When one comes to explaining this law in terms of the relation between mind and body there are an indefinite number of possible physiological (even cortical) conditions which might be selected as the basis of the conscious series of events. Many chemical processes proceed in accordance with a logarithmic law, and the mere similarity in the curves of the change of the conscious states

and the physiological (or biochemical) changes of state cannot therefore serve as a guide. And yet this sort of thing is constantly being done.

The best that statistics can do is *suggest* a *possible* causal relation, but this suspected causality must be tested by Mill's methods applied to individual cases. We must always remember that we use statistical methods when direct and detailed causal analysis is impossible, and that when we have a knowledge of the causes in their quantitative aspects and can control them to suit our purposes we do not need to resort to statistics.

Chapter VII

THE NATURE OF EXPLANATION

IN the preceding chapters we have argued that we get our hypothesis which explains by finding something in our past experience similar to the new situation, which enables us to assert that to the extent that the novel phenomenon resembles the already familiar phenomenon to that extent the new cause will resemble the old cause. In other words, in so far as we can explain we classify the new phenomenon in some old group of things or behaviors with which we are already acquainted. If this phenomenon is so unprecedented that it refuses to fit into old classes we create a new class. Thus the class of mental phenomena is not (at present at least) reducible to the class of physiological processes, so we make a new and distinct class for these conscious processes. But as the new phenomenon in time becomes more familiar we in turn "explain" the separate acts of an individual by referring them to some appropriate species of the genus of psychic phenomena. Thus we may explain the conduct of a certain individual by saying that he does what he does or thinks what he thinks because he has a "strong will," or a "good memory," or an "active imagination," and so on. These terms do not give us genuine explanations, but are classifications. Such terms designate the fact that we want to place the man's actions in its proper category of experiences or responses, and that for the present we do not propose, or are unable, to analyze further the nature of this conduct of mind and "reduce" it to some other phenomenon. But even if we did take

this additional step, it would be but a further refinement in our classification.

As we have suggested on several occasions, it is really impossible to explain *why* a thing happens. For one thing, to "explain" any single item or event in this cosmos we would really have to understand everything, for eventually the entire history of the cosmos is (theoretically) implicated in any given event. This is what is meant by the "infinite regress" of causes. However, as we have also insisted, for practical purposes an investigation of the entire cosmos is unnecessary to "understand" any event or effect. For ordinary purposes our analysis need go no further than an isolation of those variables which, by getting rid of them or by introducing them (or it), we can eliminate or produce (depending on our purposes) what we regard as the effect.

The contrast is sometimes made between *verbal* explanation and *scientific* explanation. Verbal explanation is not supposed to be genuine explanation. According to the customary interpretation, a verbal explanation merely substitutes another word (or set of words) for the phenomenon in question. It is merely rebaptizing the thing to be explained. As illustrations of verbal explanation we have such terms as "instinct," "catalysis," "friction," "transparency," "chemical affinity," and the like. If you ask, Why does an animal behave as it does? the answer given by verbal explanation will be, Because it has a gregarious instinct. If you ask, Why can you see through window glass? the answer is, Because it is transparent. If you ask, Why do carbon and oxygen unite in the process of combustion? the reply is that objects burn because carbon and oxygen have a *chemical affinity* for each other. And so on. But as everyone can see, to say that men live in groups because they are gregarious is only to rename the phenomenon. The same is true for

each of the other "explanations." This type of explanation is no better than that offered by the Medievals who explained that drugs put you to sleep because they have "dormific" powers. This explanation is on a par with the famous formula that *nature abhors a vacuum.* As John B. Watson has argued, it is just as sensible to say that a rubber ball rebounds when it is thrown against a wall because it has an "instinct to rebound," as it is to say that a man behaves as he does because of the influence of some instinct or other.

But does this mean that these terms are therefore useless and ought to be thrown out of science, as Watson proposes to do with the term instinct? Not at all! These terms are useful as labels, as tags designating certain types or classes of behavior. If we do not allow our critical faculties to be lulled to sleep by their use these words are very useful and do no harm.

But what then, it is asked, is *real* explanation? The answer is that aside from good classifications there is no genuine explanation. What science does in analyzing situations and searching for causes, for scientific explanations, is merely to refine and to improve the classifications. A scientific explanation of *why* we see through glass would tell us what it is about the nature of glass (composed mainly of silicon dioxide) which permits the transmission of certain kinds of light rays. But of course we can always ask the next question: Why do some kinds of molecular patterns *transmit* light (thus making windows "transparent") and other kinds of molecular patterns absorb or reflect light (thus producing "opacity")? In the last analysis we can only say that the facts are what they are; we point to the phenomenon, and classify other things with respect to their inclusion in or exclusion from this class of behavior. To repeat, therefore, we say that explanation is merely exhibiting a new phenom-

enon as a special case of a previously established and usually more familiar type of phenomenon or causal law. But a causal law is a generalization of an inductive type expressing a uniform correlation between classes.

From this point of view most of what passes as explanation is really "passing the buck" from one field of science to another. Sociology explains its phenomena when it exhibits group conduct as special illustrations of the laws of psychology. Psychology of a monistic sort explains its phenomena when it finds the physiological conditions of those phenomena. But if your psychological theory is of a dualistic sort the series of mental phenomena is held not to be reducible to the series of physiological events, so that that type of explanation is spurned. Physiology, in turn, explains the data of its science by exhibiting organic behavior as special applications of the laws of biochemistry. This in turn reduces to physical chemistry and physics. Eventually the physicist inherits all these problems. And now how does the physicist proceed? To whom can he pass the buck?

The phenomena of physics are supposed to be due to the characteristic modes of behavior of the ultimate constituents of matter, *electrons* and *protons,* which are negative and positive corpuscles or charges of electricity. The laws of these supposedly ultimate units are (*a*) that like-signed particles attract each other (or "tractate"), and (*b*) unlike-signed particles repel each other (or "pellate"). But, be it noted, "attraction" and "repulsion" are here only names for the fact that some particles move towards each other and other particles move away from each other. No one knows *why* these particles do this. Still further, no one ever will know the "real explanation" for the behavior of the ultimate constituents of matter. After all, attraction and repulsion are rela-

tive terms. It is just as sensible to say that two particles move towards each other because they are pushed together by external forces as it is to explain their behavior by forces acting between them. Some theorists have attempted to explain the attraction and repulsion of elementary charged particles in terms of stresses and strains in the ether of space, or in terms of "lines of force" which are conceived to "spiderweb" space. But this is only removing the difficulty another step. One can always ask why these lines of force have the particular properties which are assigned to them. Ordinarily when we come to the end of our intellectual rope we can at least point to (or denote) the fact to be explained. But "tensions" and the "elasticity" of stretched lines of force are by their very nature forever beyond direct observation, and for that reason must remain hypothetical constructs. In short, all such attempted explanations are really trying to explain the more familiar in terms of the less familiar, and we have seen that the proper mode of procedure is just the reverse. The recent attempts to reduce ultimate physical phenomena to mathematics have not met with success, so that it seems to be true that the ultimate phenomena of physics are inexplicable.

Conclusions

Summarizing what we have said on the subject of explanation, we find that the idea itself has undergone an evolution. This evolution is associated with the parallel evolution of the notion of cause. For to explain a thing or event has historically meant to find the cause for the existence and nature of that thing or event.

For primitive man, as we have seen, to explain a thing was to fit it into the animistic scheme, to tell "why" it happened in terms of the purposes of the spirits which pervaded nature. Later on, with the appearance of

philosophy and early science, explanations frequently took the form of finding some "principle" (usually, though mistakenly, referred to today as "metaphysical") which was supposed to produce the effect. In more recent scientific doctrine explanation, while still a search for "causes," has been freed from subjective and anthropomorphic elements. (This is true for natural science, but the statement is questionable when applied to any science which studies human individuals.)

Explanation, then, is finding the causes of things. And to establish a causal relation now means to show how the phenomenon in question is functionally related to other preceding and concomitant things and processes. Thus any given phenomenon appears as a center of radiation of functional relations which ultimately ramify into the entire cosmos, past and present. Fortunately, for practical purposes—prediction and control—it is usually not necessary to consider all these other conditions, for their effects are negligible. These "practical purposes" refer to the point of view we have when we make an analysis. Theoretically an event may have a number of different causes (functional relationships), but the purpose we have in mind will usually determine which of them is selected. There is here the same relativity which we have noted in connection with classification. This association is more than accidental, for a law, as a statement of an invariable functional relationship, expresses a constant correlation between classes. From this point of view the difference between a law of natural science and the superstitions of primitive man is the difference between a natural classification and an artificial classification.

Chapter VIII

HUMAN HISTORY AND INTELLECTUAL PROGRESS

THE question is sometimes discussed as to whether and, if so, in what sense the human race has made progress. In this, our last chapter, it is appropriate to consider this problem of intellectual progress, particularly as it is related to the reorganization of our systems of knowledge.

In surveying the various attitudes which are taken, we find, on the one hand, those cynics who hold that the only thing history teaches us is that history teaches us nothing. According to those who accept this view, each generation has to learn anew the lessons of life, so that the stupidities of individual and social conduct constitute an ever-recurring cycle.

There are those, on the other hand, who point out that there is usually a credit side to every ledger, and that when we come to examine this side of the account we find assets which serve in a measure to balance the liabilities which are recorded in the book of human history. This view is certainly in line with what we would like to believe. History would indeed present a sorry spectacle if the human race were not, in some sense or other, wiser today than it was thousands of years ago. If intellectual progress be a vain delusion then mankind is surely an object of pity. If modern scientists, for example, are no closer to the truth about nature than was primitive man, then human beings may well consider the question of whether anything in this world is worth the time and effort which it costs.

There is another angle to this matter which we must

also take into account. It may also be argued that an optimistic attitude towards the belief in intellectual progress has a very real pragmatic sanction. Here we apply to ideas and beliefs the old maxim, "By their fruits ye shall know them!" This practical justification rests on the general principle that the belief in the truth or validity of a proposition or ideal sometimes brings about the actualization of what is asserted by that belief or ideal. In his famous essay on the "Will to Believe," William James argued that if the belief that you are a responsible moral agent actually leads you to hold yourself responsible for your acts, then you make the belief that you have "freedom of choice" true by that very faith which is in you. More recently, but in a somewhat similar vein, Professor T. V. Smith[1] has attempted to give the notion of social equality a pragmatic sanction. He lays down the proposition that *men are what they function as*. If we function as members of a commonwealth equal in the sight of the law, then to this extent that we treat each other as equal we *are* equal. Here, again, the belief in the truth of the proposition tends to make valid that which the proposition asserts. In the same way one might argue that the belief in intellectual progress justifies itself, if the belief tends to create or bring about the facts which are asserted by the doctrine itself.

Of course, as the reader will readily observe, not all beliefs are on the same level in this respect. For example, merely believing that you have sufficient funds in the bank to cover outstanding checks does not alter the real facts, which may be quite the reverse. Again, it is hard to see how the fervent belief in immortality tends to produce the fact it alleges. The truth of the matters which science decides, and which are covered by the three classes of propositions of Bertrand Russell which we discussed in

[1] In his book, *The American Philosophy of Equality*.

the First Chapter, Part I, is quite independent of our hopes or fears.

This brings us to the point that while the belief in progress may tend to create progress, just as the belief in the necessity for change may accelerate the processes of change, nevertheless all types of progress are not on the same level. There are various possible types of progress, such as moral, social, intellectual, biological, and so forth, and it is possible that we can, by taking thought, influence the rate of progress or retrogression in one or a few of these without being able to do the same for the others.

Even though we agree that the human race has advanced, we find that when we try to state just what it is which places modern man upon the mental shoulders of his earlier ancestors we are at a loss for precise principles. Thus, assuming that there has been intellectual progress, to what are we to ascribe this advancement? There are several schools each with its own answer to this question. There are those who say that so far as innate capacities are concerned, there is no reason to believe that the average man of today is any better endowed than his forefathers of thousands of years ago. Those who hold to this view say that whatever superiority modern man possesses is due to the superior social environment, the precious cultural heritage, which we now own. Those who hold to this view are called the environmentalists. The hereditarians, on the other hand, take the stand that intellectual progress, wherever it has occurred, has always been dependent upon a superior biological stock, and that the ups and downs of civilization are an expression of the variability of innate qualities in the different races and ages of men. Fortunately for us we do not have to decide this question of whether our superiority, such as it is, consists in an improvement of

our chromosomes or in the social and institutional influences.

Here we are concerned only with pointing out one specific mark of superiority, which seems to be a symptom of some sort of intellectual progress. In all seriousness the idea is put forth that one great distinguishing feature of the present age is to be found in the fact that we no longer waste time in trying to solve some of the meaningless problems which puzzled our ancestors. Part of the wisdom of life which we have gained consists in the knowledge of what kinds of questions are sensible and what kinds are foolish or meaningless. There are several ways by which a meaningless question may be recognized. For the present we may note that one criterion, though not an infallible one, for recognizing a meaningless problem is given in the test of whether the problem has a solution. A meaningless question is one to which there is no sensible reply.

And now, having said that much, we must proceed to qualify our statement by adding that there is a certain relativity about this test. We can probably all agree that the problem of trying to square the circle is an illustration of an insoluble, and hence, by our test, a meaningless problem. The same may be said of the attempt to produce perpetual motion. But now let us take another case. A historian tells me that his students find it very interesting to discuss the question of whether any of the wars which the United States has had might have been prevented. Certainly this is an insoluble problem. Is it therefore a meaningless problem? I think we must agree that this question has meaning. But this is because we implicitly assume that we might, by using the right kind of technique, actually find the solution to the problem. So that our original statement, that a meaningless problem is one which has no solution—either now or at some

time in the future—still remains logically intact. This situation calls to our attention the fact that we must recognize that questions which seem meaningless or meaningful in one age may become the exact opposite in some other period of cultural history. In the light of new discoveries and experiences propositions may lose or gain in significance. There appears to be such a thing as an intellectual climate for particular ages or social groups, and if we are acclimated to one kind of intellectual atmosphere this may unfit us for another. At the present time the methods and ideals of medieval thought do not appear to most of us to be commendable, but who knows how our own enlightened atmosphere may appeal to the intellectual nostrils of future generations? But we must not digress.

If we are to maintain the thesis that the human race has made permanent intellectual progress we must assume that we have not been going around in circles in our mental evolution. This view does not make it necessary for the line of progress to be straight; speaking figuratively, we might say that it could, for example, be wobbly. But in any case, the general trend must be upward and onward. This implies that we have forever abandoned as meaningless some, but not necessarily all, of the problems with which previous ages have been preoccupied. It is hard to anticipate how many riddles and questions one could collect which have attracted the interest and the intellectual efforts of men, to which we no longer give serious consideration. But it may be pointed out that the seven world riddles to which Emil du Bois Reymond [2] responded with his famous *ignorabimus*

[2] For a statement and discussion of these riddles see Ernst Haeckel's once popular book, *The Riddle of the Universe*. One of these riddles was that of the nature of the interaction of *force* and *matter*. It is interesting to note that Einstein abolishes this problem in his reinterpretation of nature as summed up in the theory of relativity.

would not all appear in our list of meaningless problems. We must also note that some kinds of problems and questions do not call forth much serious consideration at the present time, not for the reason that these questions are meaningless, but because they demand for their solution knowledge which we do not now have and which we are not likely to have for many years to come. Here we might cite the problem of the relation of consciousness to the nervous system.

Some of the foregoing illustrations we have used have been debatable. But there are other types of questions which are meaningless because of the very way in which they are put. Any attempt to answer such a question begs the question in some way. To be specific, the question of *why* there is a universe may be considered. This age-old question of what is sometimes called the "riddle of existence" is one which has worried many persons. Perhaps the reader, like the writer in the days of his youth, liked to make himself dizzy pondering over the problem of what would be if nothing was. Various thinkers throughout the ages have actually attempted to solve these puzzles. But it seems no exaggeration to state that today no disciplined thinker takes this riddle of the sphinx very seriously. Certainly all intellectually disinterested persons who consider the problem recognize that would-be solutions to this problem are specious solutions. One famous answer to this question, an answer with which we are all familiar, is summed up in the statement that the universe is because God made it. This supposed solution is hardly less naïve than that provided by Hindu mythology, which explained what held the world up by stating that an elephant standing on the back of a tortoise supported the world—without telling us what held the tortoise up! The child's question, of course, can always be asked: But who made God? Very

obviously, there is no answer to this question. The simple fact is that no one knows, ever has known, or ever will know, why the universe is.

The question of why there is a universe is a meaningless question. It is nonsensical, yesterday, today, and forever. Its seeming meaningfulness arises out of the apparently legitimate (though really invalid) extension to the whole of the universe of a type of question which is meaningful when applied to some things in that universe. A little reflection on this matter will certainly make that point clear.

Assuming, then, that we can never know why there is a universe, does this mean that every question concerning the world as a whole is therefore meaningless? This does not necessarily follow. It is true that we will never know *why* reality is. As thus stated, the "riddle of the universe" is incapable of solution. But it is entirely possible that by patient investigation we may eventually discover *what kind of a universe it is which we are living in.* The sciences, philosophies, and religions, each in their own ways, have attempted to provide an answer to this problem.

It is quite likely that these answers are not ultimate or final. New, and perhaps more surprising, solutions to this problem may be presented in the future. In this, the early infancy of the human race, we cannot hope for any complete or adequate statement of the kind of universe we are living in. But we may well hope that further progress will bring the kind of convergence of evidence which is necessary if we are eventually to prove the truth of one of the contending theories of the nature of things, with the consequent elimination of the rival hypotheses. It would be arrogance on the part of the writer to attempt to anticipate the characteristics of such a view. What we are here concerned with is to reiterate the

rather obvious fact that the universe is, and that no sensible question can be asked as to the *why* of it. All that we can say on this matter was said by Thomas Carlyle, who, when told that a well-known authoress "accepted the universe," replied: "Egad, she'd better!"

Investigators working in the field of the natural and the social sciences are sometimes impatient with this type of inquiry. They may be disposed to raise the question, "What difference does it make whether we have a theory of symbolism and a criterion of meaningfulness?" In reply the logician may point out that the view that while these problems may be of theoretical interest they are without practical significance overlooks the fact that if history teaches us anything it is that theory and practise are so intertwined that progress in one calls for, and creates, progress in the other. It may be pointed out, moreover, that it is not unlikely that scientists have wasted much time in trying to solve meaningless problems. If more time had been given to the question "when is a problem meaningful?" much wasted effort might have been saved for the solution of more intelligible problems. In his book, *The Logic of Modern Physics,* Professor Bridgman suggests (p. 30) a list of questions which might be considered as examples of meaningless questions. Here are a few selected from his list:

(A) Was there ever a time when matter did not exist?
(B) Why does time flow?
(C) May space be bounded?
(D) Is a universe possible in which $2 + 2 \neq 4$?
(E) Why does negative electricity attract positive?
(F) Why does nature obey laws?
(G) Can we be sure that our logical processes are valid?

According to Professor Bridgman a question is meaningless when the operation necessary to determine the answer to the question cannot be performed. With this

test in mind the reader can no doubt find a number of additional candidates for a list of meaningless problems.

The foregoing discussion indicates how important the problem of meaning is for science. It is quite certain that one of the next great steps in the advance of science will consist in a consideration of just such logical problems as we have touched upon. What is a problem? When is an alleged scientific problem a genuine problem? What is a solution to a problem? And how can we differentiate between a real and a spurious solution to a theoretical problem?

These are questions which come out of logic but invade the field of science. As science gives us new facts it forces us to reorganize old beliefs. Thus as time goes on we outgrow old problems and create new ones. And this process has continued and will continue throughout the history of man's intellectual progress.

The problem of explaining the rise and fall of problems is an intricate one. This is no less than the problem of explaining human history and social change. Difficult as it is, the question is sufficiently important to merit some discussion in a book on logic. Let us therefore turn our attention to this matter.

Cultural Evolution

There are several different theories of the causes of social change. Some authorities adhere to what is called the materialistic interpretation of history. According to this view, associated principally with the name of Karl Marx, the real moving forces behind history are economic needs. "Ideas" play a minor rôle in the progress of the world, according to the economic interpretation of history.

Another conception of history, called the idealistic interpretation, stresses the importance of ideas as the

cause of intellectual and social progress. When Emerson said, "Beware when God lets loose a thinker of this planet!" he was epitomizing this interpretation. When Thomas Carlyle attempted to explain the French Revolution in terms of the dominant figures—Robespierre, and others—he was aligning himself with this view.

And what attitude does logic take towards this dispute? In the first place, logicians will here, as usual, point out that you must carefully define your terms before you can hope to settle such an argument. What, exactly, do we mean by an "economic" force? And what do the Idealists mean by an "idea"? Is an idea something which has no relation to our desires, our "economic" wants? Or do all our cravings have some degree of ideational content? Thus we see that this problem soon drags in many disputed matters of theoretical psychology.

Our own view, as set forth in previous chapters, has advocated the idea that reason and emotion play into each other in such an intimate way that they can be separated only through a process of abstraction. This apparently commits us to the view that human history is partly rational and partly irrational. We must conclude that in history, as in the individual, various forces are at work, and that an adequate explanation of either will admit the working efficacy of each type of agency which can be established to be operating as a causal agency. This still permits us to conclude that in many cases where critical points occur in the history of human culture new ideas (intellectual forces, so-called) are responsible for the new movement, the new social vector. How this idea originated is indeed an intricate problem. But that new ideas do appear seems unquestionable. And since eventually the individual is the source of the new idea, the social mutation, we are probably safe in concluding that in most cases changes in the course of history are produced by

exceptional individuals. So far as cultural progress is concerned, history is but the lengthened shadow of great men, as someone has said.

We must recognize that when intelligence first appeared, in the early morning of the mind, ages ago, a new tool was introduced into the biological struggle for existence. Whatever the time and the place of this tendency to reason things out, to explain and predict, it must be admitted that from that time on the new instrument, involving memory and creative imagination, began to replace the older factors of survival; sharp teeth, powerful muscles, mere size and sheer weight yielded gradually to the most mighty and subtle of nature's creations, the cerebral hemispheres of the big-brained mammals.

One can indeed become rhapsodical on this theme of the creative power of the mind. From the point of view of an idealistic interpretation, the history of civilization is the record of the march of the mind through time. It is the story of minds vibrating with the passion to create; of minds feeling the need to give expression to the yearning to mold the world a little closer to ideals; of minds fired with the visionary's zeal to make a funeral pyre of an unjust social order and replace it with Utopia; of minds which, tentatively exploring the shoals of the universe, are eventually drawn outward to depths which do not answer to the soundings of our intellectual plumb lines.

When we look at the panorama of human culture from this point of view we rise above the narrow limits of a fanatical moralist's perspective. We survey the rise and fall of cultural complexes from the esthetic point of view. Such a disinterested interest in the dreams and myths, the customs and conventions, of the human race is more akin to the ethically neutral attitude of the anatomist and the morphologist, who study dispassionately the

structure and the function of the organs of living creatures.

One writer has already suggested the desirability of a new science of the type described in the above paragraph. Dr. Smith Ely Jelliffe [3] proposes the term *paleo-psychology* to designate this new psychological-cultural investigation. This science, studying what may be called the fossilized forms of mental evolution in a fashion analogous to the study of paleontology, which investigates the structures of ancient and extinct animals, differs from its sister science in that it need not limit itself to the petrified forms of human thought of past ages, but can also study existing types of idea-structures as survivals and reconstructions of more primitive systems of beliefs. Paleo-psychology thus includes the comparative anatomy of ideas.

As illustrations of fossils of mental evolution the following may be suggested:

(A) The Notion that *Matter is Evil.*
(B) The Doctrine of *Faculty Psychology.*
(C) The Doctrine of the *Divine Right of Kings.*
(D) The *Social Contract Theory* of the Origin of Society.

No doubt many other fossils might be added to the list, but it is beyond the scope of the present work to go into this matter in detail.

Conclusion

Looking back over our survey of the mind in action, we recall what an important rôle intelligence has played in the initiation of human progress and the creation of civilization. We note the social factors which foster the continued use of this most valuable of nature's gifts to man. We observe how science, as a technique for

[3] In his Introduction to Dr. Herbert Silberer's *Problems of Mysticism and Its Symbolism.*

mastering the world of nature, serves not only human practical purposes, but also satisfies in some degree our craving to understand and explain. But we also discover the present limitations to scientific comprehension. Particularly we note how little the human individual understands himself, and why in this field so much remains to be done.

We have discovered how fearfully potent new ideas may be. But as we have also seen, science can tell us little about the genesis of new intellectual creations out of that vibrating mass of protoplasmic stuff which is our brains. Perhaps here, in the inner laboratory of the mind, something occurs analogous to what happens in the chemist's laboratory. The chemist puts certain proportions of hydrogen and oxygen together in a flask. Nothing happens. He then passes an electric spark through the gases. There is a flash—and the two unite to form water, a liquid, a creative synthesis. In the same way, the mind, that supreme alchemist of nature, passes her sparks through the old mixtures and transmutes them into new fabrications. What happens no one knows. But we do know that this subtle distiller of new cerebral essences is more efficient in some of us than in others. To learn the secret of this sublimation and to use the knowledge thus gained to create novel intellectual syntheses—that is as noble a goal as the human mind has yet conceived!

QUESTIONS AND EXERCISES

Part I and Part II

The Nature of Logic

1. What is logic?
2. Why is logic called a normative science?
3. How does the logician's interest in reasoning differ from the psychologist's interest?
4. How do the norms of logic differ from the norms established by the mental tests given by the psychologists?
5. How does logic differ from mathematics?
6. What does logic have in common with mathematics?
7. What does logic have in common with ethics, and how does logic differ from ethics?
8. What was Socrates' theory of the relation of morality and intelligence?
9. Trace the history of logic, stating the main contributors and their contributions to this development.

Logic and the Life of Reason

1. What is the importance of good thinking to: (a) a lawyer; (b) a business man; (c) a teacher; (d) a doctor?
2. What is logical consistency? Is it possible to be completely consistent in an evolving world?
3. Under what conditions is reasoned conduct more efficacious than instinctive action? When is automatic conduct more desirable than reasoned behavior?
4. What is prejudice? How is it related to emotion? How is prejudice related to logical consistency?
5. What ought to be the proper relation between reason and emotion? Are the two inevitable enemies of each other? Can these two ever be completely separated? What does modern psychology have to say about faculty psychology?
6. Do we owe any respect to authority and tradition? To what extent ought we respect the customs and beliefs inherited from the past? Why is customary or conventional judgment of a certain type of conduct often right?
7. What are some of the reasons why men fear changes in the laws of the state?

Part III

Words and Definition

1. Bring to class two technical definitions you have come across in your other studies. Can you redefine the terms in such a way that some meaning is conveyed to other persons who are not experts in the field in which these definitions are employed?
2. Define the terms fact, knowledge, belief, truth and falsity, bringing out the relations between them.
3. State the requirements of good definition.
4. In the light of these requirements criticize the following definitions:
   (a) Intelligence is the thing that intelligence tests measure.
   (b) Belief is the consequence of an association of ideas.
   (c) Logic is a normative science.
   (d) Matter is the stuff of which things are made.
   (e) A religion is that which satisfies the highest needs of man.
   (f) A sensation is a modification of consciousness.
   (g) A liar is a person who wilfully misplaces his ontological predicates.
5. Give a satisfactory definition of each of the following:
   (a) Liberalism (d) Propaganda
   (b) Intelligence (e) Public Opinion
   (c) Common Sense (f) Democracy
6. Are the following terms defined by postulation or by induction (inspection)?:
   (a) Point (e) Beauty
   (b) Book (f) Energy
   (c) Parallel lines (g) God
   (d) Soul (h) Temperance
7. Look up the meanings assigned to the following words in different periods in the history of thought:
   (a) Force (c) Mind (e) Morality
   (b) Atom (d) Knave (f) Justice

The Interpretations of Propositions

1. Find one meaning in which each of the following propositions is true and another in which each of them is false:
   (a) We should live according to nature.
   (b) Men cannot be made moral by legislation.
   (c) The wages of sin is death.
   (d) All men are created free and equal.
   (e) People like to be humbugged.

2. Find the questionable word in the following arguments and substitute the right word:
   (a) Transitory things should not be taken seriously;
   Life is transitory;
   Therefore, life should not be taken seriously.
   (b) Self-denial is the essence of virtue;
   Therefore, a man who is naturally kind and honest should practice the opposite.
   (c) Discontent is an essential condition of progress;
   But chronic discontent means chronic sorrow;
   Hence continued progress requires chronic discontent and so chronic sorrow.
3. Do you accept the following arguments? If not, how would you criticize each of them?
   (a) Improbable events happen almost every day;
   Events which happen almost every day are probable events;
   Therefore, improbable events are probable events.
   (b) No cat has nine tails;
   One cat has one tail more than no cat;
   Therefore, one cat has ten tails.
   (c) In going around the world westward we keep gaining time, and the whole trip would gain us one day; therefore, if we could make the complete journey in 24 hours, it would really take us no time at all.

Immediate Inference

1. What is meant by the logical form of a proposition?
2. Put the following propositions in logical form:
   (a) It is snowing.
   (b) Socrates believed that no one would knowingly do wrong.
   (c) My mind to me a kingdom is.
   (d) Some vertebrates are primates.
   (e) He little knows you, who calls you a coward.
3. Explain the square of opposition.
4. What is meant by immediate inference?
5. Give the contradictory and the contrary (or sub-contrary) of each of the following:
   (a) All men desire wealth.
   (b) All is not lost.
   (c) Pain alone is evil.
   (d) Not all men are brave.
   (e) No coward need apply.
   (f) Some knowledge is not of any value.
   (g) Anybody but a fool can see that.

6. Define the processes of obversion, conversion and contraposition.
7. Give the obverse and whenever possible the converse and contrapositive of the following propositions:
   (a) Nothing actual is impossible.
   (b) Phosphorus does not dissolve in water.
   (c) All substances which are material possess gravity.
   (d) All freed criminals are dangerous to society.
   (e) Some metals are conductors of electricity.
   (f) No true patriots are traitors.
   (g) Some pretty girls are not intelligent.
8. Give all the immediate inferences you can from the following propositions:
   (a) All trespassers will be prosecuted.
   (b) Gentlemen prefer blonds.

Syllogistic Inference

1. State the rules governing syllogistic inference.
2. What is meant by the figure and the mood of a syllogism?
3. State whether the following arguments are valid or invalid, and if invalid why. Test also by diagram.
   (a) He does not despise money, for he is a sensible man, and no sensible man despises money.
   (b) Rational beings are accountable for their actions;
   Brutes are not rational beings;
   Therefore, brutes are not accountable for their actions.
   (c) Every honest man attends to his business;
   This person attends to his business;
   Therefore, he is an honest man.
   (d) All good citizens are voters;
   This man is not a good citizen;
   Therefore, this man is not a voter.
   (e) No persons who are destitute of imagination are true poets;
   Some persons who are destitute of imagination are good reasoners;
   Therefore, some good reasoners are not true poets.
   (f) Utilitarians are hedonists;
   Practical men are utilitarians;
   Therefore, practical men are hedonists.
   (g) Learned men sometimes become mad; but as he is not learned there is no danger for his sanity.
   (h) Books are human productions;
   All human productions are liable to error;
   Therefore, books are liable to error.

4. Give the rules governing correct inference in the hypothetical syllogism.
5. Take several of the foregoing categorical syllogisms and convert them into hypothetical arguments, stating what fallacies, if any, are involved.
6. Give the valid and invalid forms which can be obtained from the following:

   If the gaps in the fossil remains of the animals of past ages can be filled, the doctrine of evolution will be more firmly established.
7. Explain and illustrate the disjunctive syllogism.
8. Exhibit the way in which the dilemma is a combination of the hypothetical and disjunctive modes of argument.
9. Examine the validity of the following arguments:
   (a) No strike but injures trade and consequently impoverishes the country.
   But this is to diminish the means to happiness.
   And as all that diminishes the means to happiness is to be condemned,
   We must therefore condemn strikes.
   (b) It is not right for you to devote all your time to archæological research, for if all men did so the business of the world could not go on.
   (c) Carbon, which is one of the main sources of the nourishment of plants, cannot in its simplest form be dissolved in water, and cannot therefore be absorbed in that form by plants, since the cells absorb only dissolved substances. All the carbon found in plants must consequently have entered them in a form soluble in water; and this we find in carbonic acid.
   (d) To allow every man unbounded freedom of speech must always be, on the whole, advantageous to the state; for it is highly conducive to the interests of the community that each individual should enjoy a liberty perfectly unlimited in expressing his sentiments.
   (e) A successful author must either be very industrious or very talented; Gibbon was very industrious; therefore he was not very talented.
   (f) Of university professors some are good teachers and some are zealous investigators.
   A is an excellent teacher;
   Therefore, he is not a zealous investigator.
   (g) Nothing destitute of consciousness and reason can produce out of itself beings endowed with consciousness and reason.

The universe produces beings endowed with consciousness and reason.
Therefore, the universe is itself not destitute of consciousness and reason.

(h) All hypotheses which can be proved ought to be accepted.
This hypothesis cannot be disproved.
Therefore, this hypothesis ought to be accepted.

(i) If capital punishment involves cruelty to its victims it ought to be abolished in favor of some other penalty; if it does no good to society it should be abolished. But either it involves cruelty to its victims or it does no good to society, and hence ought to be abolished.

(j) Reply to the foregoing argument in a way which is appropriate to the type of reasoning followed.

Part IV

Mathematics and Natural Science

1. Define the term, natural science.
2. What is the difference between a natural science and the formal sciences of mathematics and logic?
3. Explain the statement that mathematics is the science of necessary inference.
4. Give an example of a chain of inferences that follow necessarily, but whose initial assumptions are false.
5. Give several illustrations of sciences that have become more mathematical as they have developed.
6. Is the statement true that modern science has abandoned deductive reasoning?

Observation and Classification

1. What is the place of observation in natural science?
2. What are the difficulties which are encountered in getting adequate percepts?
3. Show why and how observation and memory usually contain some amount of inference.
4. How does this affect the reliability of generalization?
5. Explain the tests which are used in establishing the validity of correct observations.
6. What is the purpose of classification?
7. What is the difference between a natural classification and an artificial classification?
8. What is the guiding principle in the formation of a natural classification?

9. Explain the statement that classification is the bridge between inductive and deductive logic.
10. State the rules governing good classification.
11. In the light of these rules criticize and correct the following classifications:
    (a) Schools, into technical, preparatory, professional or scientific.
    (b) Books, into those on history, science, poetry, religion, and belles-lettres.
    (c) Domestic animals, into those which are useful and those which are pets.
    (d) Games, into those which are athletic and those which are intellectual.
    (e) Religions, into monotheistic and polytheistic.
12. Divide and sub-divide: Government; propositions; books.

Induction and Causal Laws

1. What is a natural law?
2. How does a natural law differ from a statutory law?
3. Are the laws of nature ever "broken"?
4. Explain the relation between the discovery of the cause of an event and the ability of the investigator to produce that event. How is this relation illustrated in the development of physics? Of biology? Of the social sciences?
5. What is induction by simple enumeration?
6. Explain the statement that induction is a process of sampling. What is meant by the "fairness" of the sample?
7. Define the fallacy of neglected aspect. Illustrate by some argument which might be used in regard to the value of examinations or of college athletics.
8. Why was it so difficult for primitive man to avoid the fallacy of false cause?
9. Why are we entitled to judge the future from the past?
10. Why must the principle of the uniformity of nature remain a *postulate* of natural science?
11. State and illustrate each of Mill's methods of induction.
12. State the precautions which must be taken in drawing conclusions from the results obtained by the use of each of Mill's methods.
13. Explain and illustrate the following: (a) the inductive leap; (b) perfect induction; (c) plurality of causes; (d) collocation of causes; (e) infinite regress of causes.
14. Analyze the following examples of reasoning and identify the method employed:

(a) The freshwater crayfish has a sense of smell, as is proved by the rapid way in which it retreats from strong odors. Investigation led to the hypothesis that this sense was located in the antennules or smaller feelers. This was verified by observing that a crayfish bereft of these appendages did not react to a strong odor, whereas in exactly the same conditions and to the same stimulus another crayfish with its antennules intact did actively respond. (J. A. Thompson, *Introduction to Science.*)

(b) The health of the thyroid gland is of supreme importance. And we know that it depends upon the regular supply to the body of a small but indispensable amount of iodine. The discovery was made in connection with the hatching and rearing of young fishes in fresh water hatcheries. The water used was unnaturally pure, and it was noted that all the fish developed goitre. The addition of a little iodine to the water, however, led to their prompt recovery.

(c) The quantity of oxygen inhaled has a profound effect upon the amount of work that can be performed by the body, which requires oxygen to burn the carbon assimilated in the food. The amount required per minute in walking and running rises as the speed increases, and at high speeds, as in a hundred-yard spurt, the amount is ten times that required when sitting or standing still. It appears from experiments that oxygen inhaled during short periods of exercise is used to oxidize lactic acid and carbohydrates in the blood. But why is more required when the exertions are prolonged? This additional oxygen must be used in oxidizing other substances in the blood. What these other substances are may be determined by later experiments. (D. S. Robinson, *Illustrations of the Methods of Reasoning.*)

(d) Overdriven cattle, if killed before recovery from their fatigue, become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death; cocks killed during or shortly after a fight; and soldiers slain in the field of battle. These various cases agree in no circumstances directly connected with the muscles, except that these have been subjected to exhausting exercise.

(e) Germs grow faster when they get their light arranged in a certain way, according to two Indian scientists.

The two experimenters took cultures of the germs of typhoid fever and cholera, and exposed one set to ordinary light, while the second set was placed under a beam of polarized light. Polarized light differs from ordinary light in that the fronts of the light waves are all arranged in the same direction, while in ordinary light they crisscross at all angles. The germs seemed to thrive much better on polarized light than on ordinary light.

(f) It is evident that the green color of plants holds some necessary relation to light, for the leaves of plants growing in the dark, as potatoes sprouting in the cellar, do not develop this color. Even when leaves have developed the green color, they lose it if deprived of light, as is shown by the process of blanching celery and by the effect on the color if a board has lain upon it for a long time. (Coulter.)

(g) Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyl of carbon dioxide under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions of the leaves of an illuminated plant were covered with black paper, no starch was formed in these portions.

(h) The great famine in Ireland began in 1845 and increased until it reached a climax in 1848. During this time agrarian crime increased very rapidly until, in 1848, it was more than three times as great as in 1845. After this it decreased with the return of better crops until, in 1851, it was only 50 per cent. more than in 1845. It was evident from this that a close relation of cause and effect exists between famine and agrarian crime.

Hypothesis and Analogy

1. What is an hypothesis?
2. What are the uses or values of an hypothesis?
3. How do we get our hypotheses?
4. State the requirements of a good hypothesis.
5. What is the relation between an hypothesis, explanation and induction?

6. Why are we justified in preferring the simpler of two hypotheses, other things being equal?
7. In what way may the law of parsimony be abused?
8. Explain the method of plural hypotheses.
9. In what respects does the doctrine of evolution provide a better explanation of the presence of living creatures here on earth than the special creation theory?
10. What is the general method for testing an hypothesis?
11. What is meant by analogy, and how is it related to hypothesis?
12. Why are analogies frequently untrustworthy?
13. Analyze the following arguments and state whether the analogies are valid or invalid:
    (a) No body can be healthful without exercise. And certainly to a kingdom or a state, a just and honorable war is the true exercise. A civil war, indeed, is like the heat of a fever; but a foreign war is like the heat of exercise, and serveth to keep the body in health. (Francis Bacon.)
    (b) We should think it a sin and a shame if a great steamer, dashing across the ocean, were not brought to a stop at a signal of distress from a mere smack. . . . And yet a miner is entombed alive, a painter falls from a scaffold, a brakeman is crushed in coupling cars, a merchant fails, falls ill, and dies, and organized society leaves widow and child to bitter want or degrading alms. (Henry George.)
    (c) If a merchant commenced business without any knowledge of arithmetic and bookkeeping, we should exclaim at his folly, and look for disastrous consequences. Or if, before studying anatomy, a man set up as a surgical operator, we should wonder at his audacity and pity his patients. But that parents should begin the difficult task of rearing children without ever having given a thought to the principles—physical, moral, or intellectual—which ought to guide them, excites neither surprise at the actors nor pity for their victims. (Herbert Spencer.)
    (d) Just as the thirteen colonies came together to form the United States of America, so the nations of the world will eventually unite to form a world state.
    (e) After Lavoisier had recognized the analogy between the processes of combustion and respiration, the idea naturally suggested itself to his mind that the heat observed in the animal organism must have the same

origin as that which is evolved during combustion; and, afterwards, in a paper published jointly with Laplace, these two illustrious philosophers distinctly stated it as their opinion, based upon experiment, that the sensible heat of the animal organism is the combustion heat of the carbon and hydrogen which, in the form of food, are burnt in the body. (Hofman, *The Life-work of Liebig.*)

(f) After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of thunder and lightning were the same as those produced by electricity. Lightning travels in a zigzag line, and so does an electric spark; electricity sets things on fire, so does lightning. Animals can be killed by both, and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another? (Buckley, *A Short History of Natural Science.*)

(g) We may observe a very great similitude between the earth which we inhabit and other planets. From all this similitude, we may infer that these planets are probably, like our earth, the habitation of various orders of living creatures.

(h) The atom may be compared to a small solar system. The negative electrons correspond to the planets revolving about the sun, except that in the case of the atom the central nucleus is called the proton. Since the law of attraction is the same in both cases the orbits of the electrons will resemble the orbits of the planets in being elliptical. Thus we may speak of the astronomy of the atom.

PROBABILITY AND STATISTICS

1. What is the connection between induction and probability?
2. In what sense is it true that you get absolute certainty only in mathematics and at best but a high degree of probability in an inductive science?
3. What is meant by calling a judgment probable?
4. How is probability related to chance?
5. Explain the following statement: The chances for its safe arrival may be as great for the letter that was lost as for any other.

6. Explain the fallacy involved in the following: I have had a run of bad luck, therefore my next venture is likely to succeed.
7. Under what conditions do scientists resort to statistical methods?
8. Explain how statistics may suggest causal laws.
9. Explain the common fallacies involved in the interpretation of statistical results.

The Meaning of Explanation

1. Trace the development of the notion of explanation from primitive man down to the present time.
2. Show how primitive man's notion of explanation is based upon false analogy.
3. Explain the other logical fallacies which fostered the superstitions of primitive thought.
4. What is meant by positivism?
5. In what sense is it true that explanation in science consists in "passing the buck"?
6. Explain the statement that the ultimate phenomena of physics are inexplicable.
7. Why may explanation be regarded as classification?
8. How are hypothesis and analogy related to explanation?
9. Why can we speak of analogy as incomplete explanation?

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